A STUDY OF BILATERAL CONTRACTS IN A DEREGULATED POWER SYSTEM NETWORK

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By

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ABSTRACT

One of the main objectives of deregulating the electric power industry is to introduce competition in the electricity business and prevent monopolies. The introduction of deregulation has, however, led to confusions in the areas of transmission network loss sharing and the responsibility of generation of reactive power. Because, under deregulation, the business and economic decisions in a power system are made by each individual vendor/utility in a decentralized manner. Each power producing entity operates on the principle of profit maximization by optimizing its production cost of real power, reactive power and the spinning reserve margin.

Two methods have been developed to determine a generator's share of transmission loss in a deregulated power system. They are: the Incremental Load Flow Approach (ILFA) and the Marginal Transmission Loss Approach (MTLA). The ILFA employs an iterative load flow technique. The MTLA finds the transmission loss share of a generator by utilizing the marginal rate of transmission loss. Both methods are very straightforward and can be implemented by an electric utility or an Independent System Operator (ISO) with little difficulty. Results obtained from both approaches agree well. The details of the two methods along with some numerical examples have been presented in this thesis.

The profit maximization objectives of any generating entity or an IPP not only depends on transmission loss allocation but also on the production levels of real power, reactive power and spinning reserve. A model for profit maximization by a generating entity or an IPP who is interested to sell both real and reactive power is developed and presented in this thesis. In many jurisdictions, a power producer has the option for selling spinning reserve in addition to real and reactive power. A profit maximization model based on the forecasted market price of real power, reactive power and spinning reserve has been developed and presented in this thesis. The model would help a producer to decide the production levels of these three commodities in order to realize the maximum profit. Zero profit conditions have been considered along with the profit maximization model

to determine the minimum acceptable price vectors of these three commodities. A small test network and the IEEE 24-Bus Reliability Test System (RTS) have been utilized to conduct studies and illustrate the concepts with numerical examples.

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$[A_p]$	=	coefficient of power loss equation in matrix form
a_{pij}	=	element of matrix $[A_p]$
B_{ij}	=	loss coefficient
$[B_p]$	=	coefficient of power loss equation in matrix form
b_{pij}	=	element of matrix $[B_p]$
$[C]$	=	diagonal matrix with elements $\left(\cos\delta_i / \lvert V_i \rvert\right)$
c_i	=	element of matrix [C]
$[D]$	=	diagonal matrix with elements $\left(\sin\delta_i / \lvert V_i \rvert\right)$
d_i	=	element of matrix [D]
$[I_B]$	=	bus current matrix
I_i	=	complex bus current at bus *i*
$[I_p]$	=	matrix of real components of bus currents
I_{pi}	=	real component of bus current at bus *i*
$[I_q]$	=	matrix of reactive components of bus currents
I_{qi}	=	reactive component of bus current at bus *i*
$[J]$	=	Jacobian matrix
K_{lo}	=	loss constant
G_a	=	active power generation of Generator A
$Load_{a,}$	=	real power demand of customer A
$Loss_{a,}$	=	share of real transmission loss of Generator A
M_a	=	share of reactive transmission loss of Generator A
$[P]$	=	real power injection matrix
$[P_D]$		real power demand matrix

P_{Di} = real power demand at bus i

$[P_G]$ = real power generation matrix

P_{Gi} = real power generation at bus i

P_k = real power injection at bus k

P_{kc} = calculated real power injection at bus k from AC load flow analysis

P_{ks} = specified real power injection at bus k for AC load flow analysis

P_l = total real power loss

$[Q]$ = reactive power injection matrix

$[Q_D]$ = reactive power demand matrix

Q_{Di} = real power demand at bus i

$[Q_G]$ = reactive power generation matrix

Q_{Gi} = reactive power generation of Generator at bus i

Q_k = reactive power injection at bus k

Q_{kc} = calculated reactive power injection at bus k from AC load flow analysis

Q_{ks} = specified reactive power injection at bus k for AC load flow analysis

Q_l = total reactive power loss

$[R]$ = matrix of real components of the bus impedance

r_{ij} = element of matrix [R]

S_i = complex power injection at bus i

S_l = total complex power loss

U_1 = coefficient of incremental real power loss expression

$[V_B]$ = bus voltage matrix

V_k = voltage at bus k

W_1 = coefficient of incremental reactive power loss expression

$[X]$ = matrix of reactive components of the bus impedance

x_{ij} = element of matrix [X]

Y_{kn} = element of bus admittance matrix between buses k and n

$[Z_B]$ = bus impedance matrix

ΔL_a = marginal real power loss share of Generator A

ΔM_a = marginal reactive power loss share of Generator A

ΔP_k = mismatch between specified and calculated real power injections at bus k

ΔP_l = incremental real power loss

ΔQ_k = mismatch between specified and calculated reactive power injections at bus k

ΔQ_l = incremental reactive power loss

ΔV_k = mismatch in voltage at bus k from successive iteration from AC load flow analysis

θ_{kn} = angle associated with the element Y_{kn}

δ_k = bus angle of bus k

$\Delta\delta_k$ = mismatch in angle of bus k from successive iteration from AC load flow analysis

μ = ratio of reactive to real power

P_{Dim} = real load demand at bus *m* which is supplied by generator connected at bus *i*

L_{im} = real transmission loss allocation for the load demand at bus *m* which is supplied by generator connected at bus *i*

T_{im} = reactive transmission loss allocation for the load demand at bus *m* which is supplied by generator connected at bus *i*

U_{im} = coefficient of incremental real power loss expression for generator connected at bus *i* and supplying load at bus *m*

W_{im} = coefficient of incremental reactive power loss expression for generator connected at bus *i* and supplying load at bus *m*

F = cost function of a generating plant

a, b, c, d, e = cost parameters of generator

λ = Marginal cost

$P_{\max}$ = maximum real power producing capacity of the generating unit

$P_{\min}$ = minimum real power producing limit of the generating unit

P'_{max} = maximum real power producing capacity of the generating unit with Q_{max}

Q_{max} = maximum reactive power producing capacity of the generating unit

Q_{min} = minimum reactive power producing limit of the generating unit

Q'_{max} = maximum reactive power producing capacity of the generating unit with P_{max}

Φ = price of real power

Ψ = price of reactive power

π = profit

ζ = ramp rate of a generator

CHAPTER 1: INTRODUCTION

1.1 INTRODUCTION

Electric utilities have been in business for more than a century. Starting from very small utilization networks, utilities have become widespread and complex in nature. Power system networks and utilities have gone through various stages of evolution since then. But it has been going through some unprecedented business changes over the last few years.

An electric power system is a composite entity with three main functional zones.

- Generation - generation is the process by which conventional fuels (gas, coal, nuclear fuel, etc.) or conventional renewable sources of energy (hydraulic energy or solar energy) are converted into the electric energy.
- Transmission - transmission is the process by which the generated electricity is delivered in bulk to different local networks through transmission lines from the generation plants.
- Distribution - distribution is the process of delivering the electric power from the local networks to the retail consumers/customers.

Operation of a power system is a set of complex activities which not only depend on the state of existing technology but also on other complex issues like economy, social advancement, environmental impact and political decisions. These factors, however, vary from country to country and so do the power system networks and their mode of operation. Every power generating installation, in general, involves an enormous amount of investment. That is why, any change in the grid network or its operation model often raises passionate debates. But, due to the demand of social and technological advancements the changes in power system networks and their mode of operation are inevitable. Change and modification in a progressive way are natural phenomena.

Electric power systems in the early days were developed on the concept of natural monopoly. Natural Monopoly occurs if the production costs decrease as output grows larger. If the cost of production vary greatly with the number of firms in an industry, then fewer firms will have lower costs than more firms. In the extreme situation, a single firm may have the lowest cost than all other firms, this condition is known as natural monopoly [1]. Power system networks and their operation are justified to be natural monopoly by conventional economic consideration [2]. But researchers proved that "It is simply not true that monopoly pricing is 'natural' result of a market merely because firms in the market exhibit decreasing costs and demand is sufficient to support no more than a single firm" [3]. This research shows the way that government regulation and state ownership can be substituted by fair competition to assure good performance and fair pricing to consumers.

Electricity has become an essential energy commodity and millions of equipment and accessories are being in use worldwide that solely dependent on electric power. Electric power system networks and their operations are different in different countries. But in many different parts of the world electric utilities are facing the challenge of transformation - from a regulated monopoly market to open competitive market. Although electric power is considered as an energy commodity to be traded in the market place, there exist a number of technical and economical challenges to be addressed first for a smooth transition from a regulated to an open market energy system.

In a monopoly system, only one utility controls all three functions of generation, transmission and distribution in one designated service area. This service area is primarily determined by political map and jurisdiction. In some cases, distribution is divided among two or more electric utilities e.g. city corporation or other private distribution companies. This conventional set-up is known as vertically integrated system.

Over the last two/three decades, technological advancements (e.g. metering, transmission interconnection, efficient generation etc.) have paved the way for non-utility generation systems (NUG). The introduction of NUGs has made the generation function very competitive and started the elimination process of the so-called natural monopoly in power system industry.

The possibility of a new competitive environment in this century-old regulated electric power industry has created enthusiasm among the researchers of various fields. The deregulation of electric power industry involves not only technical challenges but also various non-technical and economic issues. Along with the power system researchers, people from economics, finance, risk management and marketing are contributing to the process of full development of a deregulated power system network that ensure competition and fair competition.

1.2 POWER SYSTEMS IN DIFFERENT COUNTRIES

The original idea of a monopoly electric system still exists in many countries while other countries are progressing rapidly towards a fully deregulated system. Deregulation is based on the principle that all customers equally share the costs and the benefits of existing, regulated generation units. The customers will benefit from competition and fair pricing and the producers of electricity will benefit from an efficient operation open markets.

The nature of existing power systems in different countries, their operation and required modifications are very complex and cannot be described with a single standard model. Every country has its own unique characteristics ranging from social to political and their network systems had evolved based on these factors. Despite all challenges, many countries in the world have been restructuring their electric utilities under pressing internal and external influences. Countries ranging from very rich like Canada, United Kingdom to very less developed like Pakistan, Bangladesh, are restructuring their electrical grid systems at a different accelerating rate.

The state of California of the United States first introduced a Bill for deregulation of electric utility in 1996 [4]. The new law in California approved the California Public

Utilities Commission's (CPUC) plan for a state wide Power Exchange (PX) and a nonprofit, quasi-governmental organization called Independent System Operator (ISO) to control electrical generation and transmission [5]. Power Exchange prohibited utilities from contracting for power in advance. Instead, all electricity were required to be bought and sold by Power Exchange in day-ahead and hour-ahead spot markets. All generating companies (intended for California market) and distributing utilities were required to bid prices daily in the PX. The deregulation Bill allowed the PX to ensure that all utilities pay the same and the highest price offered on any given day. Power distributors are restricted to a price cap of 6.5 cents per KWHr for charging any residential consumer, regardless of the price paid for power in the state-managed spot market [4,6]. The utilities in California still own the transmission network, but they no longer operate it. Instead, the Independent System Operator (ISO) was given the responsibility of transmission network operation. But the ISO was not allowed to enter into contracts for power [7] and the independence of the operator is also uncertain as the deregulated utilities still own the transmission network [5].

The primary goal of deregulation is to remove or reduce state-control over electric/energy utilities. The Deregulation Bill did not limit the role of California Public Utilities Commission in this context and in addition two more controlling organizations were created (PX and ISO).

In a free market price of any commodity is free to rise and fall based on supply and demand. Eventually the market settles down to a stable price. If the price of any commodity is regulated and not allowed to rise with rising demand, the result will be a severe shortage. So an artificially restricted commodity prices prevent a supply shortage by leaving some producers out of the market [8]. The elimination of price control would bring such producers back into the market, increase the market supply share and reduce the market price. The retail consumers were relatively unharmed by the high price of electricity in California because of the government-mandated retail price caps. Utilities like Pacific Gas and Electric (PG&E), Southern California Edison suffered a heavy financial loss during the crisis period in 2000-2001 due to the restricted price capping policy of the California Government.

Not many generating plants, especially hydro, nuclear or fossil fuel-run units, were built in California in last decade or so, due to various reasons including the existing price cap. California however promoted the application of wind power as a form of green energy. Out of the total installed capacity of 53,742 MW, wind power supplies only 1,676 MW of electric power in California [9]. The injection of wind power is not enough to meet the rapid increase in demand for electric power due to rapid population growth and increase in power consumption per capita caused by economic/industrial progress. The California Energy Commission indicates demand of electric power increased moderately and steadily by a total of 9% from 1990 to 1999 [10].

Most of the experts suggested that imposing a retail rate cap was primarily responsible for the power crisis in California [11]. They argued that allowing utilities to enter into long-term contracts with power producers would have little effect in alleviating this crisis. They also expressed the view that different proposed remedies such as state takeover of the industry, the minimal increase in power rates, energy conservation subsidies, prohibitions of "wasteful" energy use, more vigorous wholesale price controls, or the adoption of long-term power contracts with generators would make the situation worse.

Other states in the United States are proceeding with deep caution in deregulating of their electric power utilities. Pennsylvania, Texas and Ohio are making good progress. Pennsylvania allowed customers to choose their utility from January 1999. Before deregulation, retail rate in Pennsylvania was 15 percent higher than anywhere in the United States. But after deregulation this rate has become 4.4 percent lower than anywhere in the United States. Unlike California, utilities in Pennsylvania are not forced to sell their generating capacity and are allowed to enter into long-term agreements for power supply[12].

Restructuring of electric utilities is going ahead with different paces in different provinces of Canada. With the inducement of Alberta's Electric Utilities Act (EUA) on January 1, 1996, Alberta became the pioneer in deregulating the power industry in Canada [13]. The Act has been introduced with the goal of introducing a full customer choice and service and promoting more efficient and competitive energy pricing. This would allow producers of electricity to determine their own pricing and investment policies according to the demands of the marketplace and competitive market forces, rather than allowing regulatory authorities to determine these policies for power production. The customers would choose their own retailer of electricity. Edmonton Power initiated the concept of customers contracting with their utility company as the flexibility offered by deregulation.

Under the EUA, all generators of power in the province are required to sell their energy to the Power Pool of Alberta and distributors are required to buy energy from the Pool. The provincial grid of transmission lines is supervised by an independent authority, which ensures that power generators and distributors can access the market on fair terms. Owners of transmission facilities and existing generating units must file separate tariffs (the costs and terms of service) with the Alberta Energy and Utilities Board (AEUB) for approval. The tariffs are implemented on an interim refundable basis subject to the results of a public review prior to final approval. Because distributors must pay a proportionate share of the cost of generating and transmitting electricity, they will receive a proportionate share of the total refunded amount, depending on their customer base.

Transmission and distribution systems under the EUA will remain regulated, and existing distribution utilities such as Edmonton Power will continue to provide connections to customers, and maintain their distribution lines. Starting in 1999, a limited number of bulk customers in Alberta are allowed to have direct access to the Power Pool, with customer choice available to the remainder of customers in 2001. Also starting in 2001, the legislated financial arrangements ("hedges") established under the Electric Utilities Act, which are currently regulated by the AEUB, will be replaced by long-term arrangements which ensure fair sharing of costs and benefits. The terms of these arrangements was set by an Independent Assessment Team and was approved by the AEUB in 1999. In future the province expects separate generation, transmission and distribution operations to accommodate new generation and retail operations.

Unfortunately Alberta found itself in a situation similar to California. Power price skyrocketed as soon as the market opened in January 2001. Alberta did not achieve the promised goals of deregulation instead went into power shortage from power surplus and became dependent on neighbouring provinces for power [14]. Alberta did increase its generation capacity in last few years.

Ontario is moving with caution after watching Alberta's experience with deregulation. Government of Ontario opened the energy market for competition in May of 2002 after moving back this date several times. The province's hydro electricity sector boasts a supply reserve. Ontario's power generation with a variety of hydro, nuclear and fossil fuels, would likely to protect it from over-reliance on one form of energy. In the process of deregulation, Ontario Government breaks down Ontario Hydro into generation, transmission and distribution utilities.

In Saskatchewan, SaskPower is the only authorized utility to serve the consumers within the provincial boundary. SaskPower owns and controls the power generating plants. It has evaluated cost-effective options to add new power supply because of the growing demand for power in Saskatchewan. As a part of the plan, SaskPower started purchasing from 210 MW non-utility (NUG) cogeneration managed by Meridian Cogeneration Project from December 1999 [15]. Although the wholesale market is open to competing suppliers, SaskPower, a crown corporation, still dominates the retail market.

The United Kingdom is one of the first nations to privatize its electric utility/industry. The overall privatization of its electricity industry was initiated shortly after a conservative government came to power in the United Kingdom in 1979. In July of 1989, the UK Electricity Act of 1989 was signed into law [16]. Under the United Kingdom's new approach, competition is intended to be the primary means for disciplining costs, prices, and service. The reform of the electric industry in the United Kingdom is considered as a success.

Two companies, National Power and PowerGen, controlled 75 to 80 percent of the United Kingdom's capacity under the original privatization. Nuclear plants, constituting 15 to 20 percent of capacity, continue to be owned by the government. In contrast to generation, the UK's transmission system is considered a natural monopoly and controlled by National Grid Company. Twelve regional electricity companies (RECs) own National Grid Company. Distribution of power is also controlled by these twelve

RECs. There is no obligation to supply on the part of any entity producing power, or providing distribution or transmission services.

Under the law, generating companies offer price schedules to supply power half-hourly from each generating unit for the following day. The pool price is the highest offer price accepted for dispatch. The offers with prices lower than the pool price receive this common pool price and the offers with prices higher than the pool price are rejected. Customer electricity prices declined about 15 percent during the first year of privatization. Prices were expected to rise as excess capacity was absorbed and indeed this began to occur after the first year. Subsequently, prices started to increase significantly. The expectation of rising prices may have stimulated the new capacity now under construction. Generating entities has planned to build 14 new generating plants with a capacity to 6,700 MW since 1994.

In South America, Argentina's electricity industry is divided into three sectors: generation, transmission and distribution [17]. The generation sector is organized on a competitive basis with independent power producers selling their production to the Wholesale Electricity Market ("WEM"). The generating entities can also have private contracts with certain other market participants for selling power. Transmission sector in Argentina is regulated and operated by different companies. Transmission companies provide third parties access to the transmission systems they own and are authorized to charge the generating companies for transmission access. Transmission companies are not allowed to have their own generating plants and also prohibited from distributing electricity. Distribution companies are allocated with individual geographical locations and each company works on monopoly basis for its allocated region. According to the law, distribution companies are regulated as to their rates for different types of users.

Like Power Exchange in California, in Argentina Compañía Administradora del Mercado Mayorista Eléctrico S.A. ("CAMMESA") sets the price of electricity in a spot market. Distribution companies and large users buy power from the generating entities through supply contracts or in the spot market. Large users pay for transmission access if they buy power directly from generating utilities through contracts. CAMMESA can dispatch power without any contracts among generation companies and distribution companies or large users in some cases. Consequently, a generation company's capacity may be dispatched to meet power demand of the pool irrespective of its contractual commitments. If a situation like this arises, the generation company is bound to buy or sell excess energy from or to the pool at spot prices.

The largest country in South America, Brazil maintains world's largest operational hydroelectric complex – Itaipu facility on the Paraná River whose capacity is 12,600 MW. Brazil has total installed capacity of 65.2 GW (Jan1, 1999) and 87% of it comes from hydro power. Brazil depends on coal and natural gas for remaining 13% capacity.

In Brazil, the electric power system consists of two major interconnected systems (South-Southeastern-Central and North-Northeastern) [18] and many small isolated systems in remote regions. These interconnected systems are separated and operationally independent. A government controlled holding company, Eletrobrás, is responsible for implementing Brazil's electric power policy. The company plans, finances, coordinates and supervises programs for the construction, expansion and operation of electric power generation, transmission and distribution systems. Power generation and transmission are dominated by Eletrobrás, while distribution is predominantly the domain of other companies owned by state and municipal authorities and a few privately owned utilities. Main transmission lines in Brazil are the property of Eletrobrás subsidiaries and state companies such as Cesp, Cemig and Copel. Although generation remains mostly under government control and transmission is not considered for privatization in near future, distribution is mostly in private hands [19].

On its way to full deregulation, Brazil is currently in the process of creating a wholesale energy market, roughly similar to the system Argentina has had for the past 6 years. Under such a system, generation, consumption and prices would follow free market conditions, and would allow for quicker responses to the fluctuations of supply and demand.

All over the world, individual countries are embracing a form of deregulation for their energy industry by moving towards a competitive, fair and liberalized market structure. Asia is also moving gradually towards the opening up of its energy markets influenced by the European success in deregulation of electric power system utility. World Bank and other international financial institute are guiding the developing countries in Asia and Africa on their way to full open market deregulation [20].

The power industry in Bangladesh has been running on a very old concept of operation [21,22]. The generation, transmission and most of the distribution is controlled by the state-owned Bangladesh Power Development Board (BPDB). BPDB is responsible for planning, construction and operation of power generation and transmission facilities throughout Bangladesh and for distribution in urban areas except capital city of Dhaka and its adjoining areas [21,23].

Due to a high level of distribution system loss in the distribution grid, two other government agencies have been assigned some of the distribution network responsibilities. Recently, to overcome this power shortage in Bangladesh, BPDB has started buying power from a private generating company for the first time in its history. As part of the reform program, private power policy was approved by the Government in the month of October,1996. Incentive packages in terms of tax exemption on imported capital machinery and equipment, spare parts etc. is being offered to the Independent Power Producers (IPP). The following two distributing agencies are primarily responsible for power distribution along with BPDB:

- Rural Electrification Board (REB): REB, established in 1978, is responsible for distribution of electricity in rural areas through a system of co-operatives known as Palli Bidyut Samities (PBS). It purchases power from both BPDB and Dhaka Electric Supply Authority (DESA) at 33 kv.
- Dhaka Electric Supply Authority (DESA): DESA, established in 1990, is responsible for distribution of electricity in entire greater Dhaka district (except rural areas which is under REB) including the metropolitan city of Dhaka. It purchases power from BPDB at 132 kv.

BPDB sells power to DESA and REB at a cost lower than the average cost of generation of energy. A separate entity called Power Grid Company of Bangladesh (PGCB) was created under donor pressure. This will control the transmission network in Bangladesh which is under control of BPDB now.

1.3 DEREGULATED ELECTRIC POWER MARKET STRUCTURE

In a conventional system, one utility or company has exclusive right of marketing electricity in one designated service area. For Example, in Saskatchewan, SaskPower produces power, carries it using the transmission network and delivers it to the residents of Saskatchewan. City corporations, in Saskatoon and Regina, act as bulk power distributors beside SaskPower. The schematic diagram of a conventional system is shown in Figure 1.1. Since the consumers do not have any choice of their utility, the market structure is very simple.

Figure 1.1: Schematic diagram of conventional power system network.

Deregulation is a dynamic concept and bringing radical changes in power system area. The primary objective of a deregulated system is to provide customers their choices of utilities. Customers would be able to choose suitable utility (or power provider) from competing utilities based on electricity rate, service or even environmental considerations. Consumer choice and changing role of electricity industry in the market place have created new challenges.

Any electric power generating utility or independent power producer (IPP) would be allowed to enter into a bilateral contract with any customer in a deregulated power system. Allowing generators or IPP to contract directly with customers creates competition on both sides of the transaction. Generators compete among themselves to supply customers. This gives customers a full range of choice of generators. Generators may charge any price the market will bear and may choose to compete not only by price but by contract duration, payment terms, type of generation and type of electric service. Thus, bilateral contracts will provide a wide range of choices to meet various customer needs.

Under bilateral contracts, which are usually long term in nature, the IPPs will be less affected by market fluctuations. But they might lose also in case of high energy demand. Using accurate load forecasting the IPPs can shield their interest in volatile situations. The demand for energy may vary in a different ways in deregulated environment than it varies now in a regulated network. In addition to the seasonal effects there would be effects of energy price, new-coming IPPs on the energy demand.

Under deregulation, transmission lines will be controlled by an Independent System Operator (ISO) or any neutral organization that would provide equal access right to all interested generating utilities. Unlike in the monopoly system, generation utilities will no longer have ownership or control of transmission and/or distribution facilities. The role of generating utilities would be restricted to selling power only. They can sell their power through bilateral contracts or power pool. Transmission service would still be regulated in order to provide fair treatment to all.

In a deregulated environment, various generation utilities would compete with each other for selling their product. They would sell their power directly to the contracted consumers or they can bid for selling power at the day ahead, hour ahead or spot market operated by a power pool. This power would be carried over the transmission network which is controlled, maintained, and repaired by a designated authority.

Large customers are very attractive to suppliers, because large amounts of energy can be sold in one place. Due to economy of scale, this makes it possible to deliver electricity at a lower per unit cost and it means the generation utility can charge a lower price and still make a profit. In rural areas, the scenario could be different because costs in serving areas of low population density are already much higher. The end result could be that large consumers in urban areas pay lower prices, while residential and rural consumers see a significant increase in their electricity bills.

Small customers like rural or the residential customers might suffer because of the deregulation unless their rates are somehow protected. The large customers will have the advantage of bargaining power because of their large demand. But to avoid this kind of situation, small customers may form cooperatives. These cooperatives can buy power from the utilities or to take more advantage they can contract with the load aggregators.

Load aggregators will be municipal or private entities that organize customers. The load aggregators will negotiate with the generating utilities in order to obtain better contracts on behalf of their clients.

The future energy market is still unpredictable and it has various options to grow in order to overcome the challenges of deregulation. The future energy market, when it takes a definite shape, might provide guidelines to the unsolved challenges. The combination of bilateral contracts, day ahead and hour ahead market or spot market would make the future energy market a complex entity than the present day vertically integrated monopoly market. The big industrial consumers, municipalities, load aggregators would compete with each other to have the best prices. The choice of green energy would also play an important role in this sector.

1.4 INDEPENDENT SYSTEM OPERATOR

In a monopoly system, a utility is responsible for delivering power to the customer and for maintaining the system security. The Load Dispatch Centre determines the load planning, scheduling and dispatching without violating the system constraints and system security.

In a deregulated system, an Independent System Operator or ISO will play the role of a supervisor for system planning and security. An ISO should perform the following duties [24]:

- Planning services
- Power market administration services
- Operations planning (scheduling) services
- Real-time operations (dispatch) services
- Metering, settlement and billing services
- Open information communication services

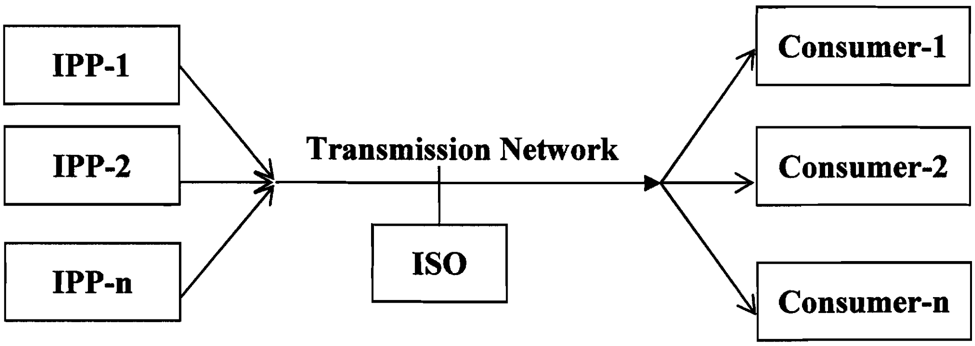

Figure 1.2: Schematic diagram deregulated power system network with an ISO.

An ISO will perform maintenance work of transmission lines on a regular basis, perform load forecast, forecast load increase in future and study the possible expansion of transmission network. It will conduct the energy auction and provide information (energy price, type of generation, type of service etc.) to all involved parties. It would schedule and dispatch load so that the system security is maintained. In case of congestion, it would figure out customer priority based on pre-determined criterion and cut-off power of the least priority customer to avoid major disaster and improve system security. An ISO will also work as a spot market for instant selling and buying of power. It would keep track of all transactions and calculate the transmission usage for each generator or IPP.

Transmission loss occurs in a system as a result of power flows through its transmission network. An ISO in a deregulated system is responsible for allocation and management of transmission loss. It buys power from spot market to make up for transmission loss.

Two different types of transactions are playing imortant role in deregulated systems. They are:

- Bilateral contracts
- Power pool

1.4.1 BILATERAL CONTRACTS

Generating utilities and customers may enter into bilateral contracts to avoid market volatility. The seller arranges the transportation of the contracted power over the transmission network. These are individual contracts and would not affect any other contract which are already in place. The concept of bilateral contracts allows the customers and generating plants to work according to their policy and does not make them dependent on everyday bid like in a power pool system. The price fixation and other services and particulars of the contract would be determined by the two parties involved in the contract. This would give them more liberty and flexibility of choice. This concept also clearly indicates whether any new transmission line should be built.

Bilateral contracts enable customers to make their best price deals for generation supply with whoever in the competitive market is most effective to meet their load demand. Allowing power producers to contract directly with customers, marketers, or retailers creates competition on both sides of the transaction. Generators compete among themselves to supply this demand. This gives customers and their representatives a full range of choices among generators. Generators may charge any price the market will bear and may choose to compete not only by price but by contract duration, payment terms, type of generation and type of electric service. Thus, bilateral contracts will provide a wide range of choices to meet various customer needs.

Customers in bilateral contracts, on other hand, have broad choices of various types of suppliers. Large customers can deal with a power producer directly or purchase power through the marketer, power broker or energy service company. Smaller customers can form load aggregators and purchase power in a similar way as the large customer. They can also get help from the energy service companies who can aggregate or sum their needs and obtain price advantage from volume, timing, load duration and other characteristics of aggregate load.

1.4.2 POWER POOL

This is the most common form of market at present due to its simple structure. Generating utilities or IPPs and customers both bid for selling and buying power at the power pool. A Power pool conducts different types of auction: day ahead market, hour ahead market, real time market etc to buy power necessary for its customers.

In a pool system, like Edmonton Power Pool and California Power Exchange (PX), both generating utilities and customers bid for selling and buying electrical power. The generating utilities do not have any target for any specific customer rather they bid for getting access to the grid. A generating utility would be out of the competitive market if its price is too high and on the other hand a customer would have no power if its offer is too low. Thus the pool fixes a single price for every hour which is determined by basic supply demand relationship of economics. All parties involved in the market have equal right to access the information regarding price and demand. A power pool system uses the existing economic dispatch procedures.

In addition to the day ahead market and hour ahead market, a power pool also operates its spot market. A spot market of electricity is somewhat different from other consumable items. Electric power is generally a non-storable item and must be consumed as it has been generated [25]. For this reason, a spot market for electric power should operate ahead of real time. A spot market can operate the bids every 10 minutes, 30 minutes or at any convenient time interval to fulfill the demand in near future.

In California, the ISO conducts 24 auctions one for each hour one day ahead. IPPs submit their bid showing the price curves over that time period and customers submit their bid in similar fashion. The ISO determines the intersection point of the aggregated demand and supply curves and set the Market Clearing Price (MCP) for every hour.

The ISO is responsible for transmission and dispatch of the contracted energy and provides information regarding pricing, transmission constraints, line losses, load distribution etc. to all bidding parties [26,27]. It is accountable for voltage suport, purchasing of spinning researve and system reliability.

One of the ISO responsibilities is to keep track of all transactions, and calculate the transmission usage for each generator or IPP. Allocation of transmission losses among the IPPs is an integral part of this responsibility. An ISO should also determine whether there exists any counter-flow in the system and if any, the sources of such counter-flow.

The generating utilities (IPPs) in deregulated environment can operate under pool system or bilateral contracts. Under a pool system, its ISO will post the hourly demand and possible price structure and the IPP will determine its optimal operating conditions based on these facts. The other factors like size and location of the plant, minimum power production requirement would also have effect on the operation of the IPPs.

Bjorgan et al [28] provide a guideline of financial contracts in an open electricity market. For the policy of selling electricity in a spot market, the effect of future contracts is evaluated. The authors show economic strategies to be followed in a spot market for selling and bidding power. Ferrero et al. used game theory for the determination of electricity price in a power pool [29]. The paper assumes that each pool participant knows its own operating cost but is unaware of competitor's cost. The Nash equilibrium point obtains the optimal price decision. Examples are given in the paper uses two participants that represents an oligopoly situation. C.Silva et al. [30] points out information suppression in deregulated environment and its effect on power dispatch. The authors argue that unwillingness to full information disclosure by competing generating entities leads to inefficient operation of a power system. Using game theory the paper presents a new mechanism that offers efficient economic load dispatch in a deregulated network in spite of the misinformation problem. The basic principle used in this paper can be summarized as – when each company acts in the best interest of its own, the outcome is efficient.

1.5 TRANSMISSION LOSS ALLOCATION

Allocation of transmission losses is a challenging and contentious issue in a deregulated system. In recent years, some works have been reported on deregulation and open access system. H.H.Happ introduces some methodologies for calculating cost of power wheeling [31]. Prior to deregulation wheeling and its associated challenges were dominating in the power sector. In addition to the existing methods for wheeling four new methodologies are presented in the paper for allocating wheeling costs among a number of wheels. Strbac et al. [32] have discussed the allocation of transmission loss by tracing the generator and load contribution to line flows. In an earlier paper Strbac et al. [33] proposed the method used for tracing the contributions. This method finds the traceable contributions of each generator and of each load to line flows instead of marginal contribution. The allocation has been proposed on the basis of maximum flows in the lines and maximum flow condition in a line has been determined by considering different load conditions and contingency. Another proposal has been made by Bialek [34,35] which used the same principle of proportional sharing at buses with a different algorithm. It determines the transmission loss by tracing back power flow from load to generator (upstream algorithm) or from generator to load (downstream algorithm). Tsukamoto and Iyoda [36] have proposed the use of game theory for solving the problem of transmission loss allocation. The cost associated with the transmission network facilities in a decentralized power system has been determined according to transmission usage pattern. The method utilizes the cooperative game theory that originally based on the rational behaviour of decision makers and traces the negotiation process by participating bodies. A generalized mathematical framework has been defined by Galiana and Ilic′ [37]. It considers various existing methods of transactions under open access system and proposes a generalized transaction matrix that takes care of all these methods. It also shows a solution for the congested conditions in networks. Transactions are showed in a virtual network and losses have been allotted to each transaction in some proportion determined by the nature of the network and by the type and level of all transactions. In another paper, Galiana and Phelan [38] show that exact loss allocation corresponding to an infinitesimal bilateral transaction is always possible. They developed a set of differential equations using a load contract matrix which yields the loss allocation for any bilateral contract. The paper shows result using a small hypothetical power system network and the solution does not include reactive transmission loss allocations. This paper only considers a deregulated power system network with bilateral contracts and cannot be applied to a system of hybrid transactions.

Cheng et al. [39] address different challenges associated with bilateral contracts in a deregulated power system network. The Authors describe modeling of bilateral contracts using a transaction matrix. A two-dimensional matrix that includes power generations and load demands is termed as transaction matrix. The study uses a 3-bus system to show how to maximize an individual contract or a generator's output using the transaction matrix. Monte Carlo methods are used for assessing operational risk of bilateral contracts in a deregulated network in the paper.

Anderson and Yang [40] offer structure for Use-of-Transmission-System charges in a deregulated environment. Instead of existing proportional sharing and circuit based methods this paper proposed a power flow comparison method to determine the use of transmission system. A 6-bus hypothetical network and IEEE 57-bus network used to demonstrate the results and compared with results obtained by existing methods. Power flow comparison method uses load flow to find a generator's contribution by super imposing the generator on the base load flow case. The difference in line flows obtained from two load flow studies is attributed to the generator's account. This method goes in sequence for each generator to calculate its effect on line flow but does not consider the sequences. Fand and David [41] discussed power dispatch issue in a power network structure dominated by bilateral and multilateral transmission contracts. A framework for price-based operation under deregulated structure was developed and a solution to optimal transmission dispatch is proposed. This paper particularly concentrates on dispatch curtail challenges with bilateral and multilateral contracts in a power system network. David also proposed a theoretical basis for coordinating pool and contracted bilateral dispatches in an open access transmission environment [42]. He also addressed the congestion management under pool and contracted dispatches. A 5-bus network was used to demonstrate different strategies for bilateral transactions presented in the paper.

A transmission loss allocation method was proposed by Expósito et al. [43] based on incremental loss factors [44]. The method presented in this paper modified incremental loss factor method and is applicable on a nodal basis. The paper shows that incremental loss factors method allocates some common loss factors to all transactions regardless of their relative sizes. The authors suggest three alternative schemes in order to allocate the mutual term to each contract – proportional allocation, quadratic allocation and geometric allocation. This paper mainly concentrates on fair splitting of the common loss factors used by one of the existing methods. Reta and Vargas [45] presented a method to trace the flow of electricity in meshed electrical networks and subsequently use this method for the calculation of transmission loss allocation. The authors argue that existing methods for tracing the flow of electricity based on proportional sharing principle are not physically and economically justified. The proposed method uses load flow and equivalent linear circuit transformation of the network to trace the flow from any generator to any designated load in a network. The procedure described in the paper determines the current fractions associated with each load and accordingly a percentage of line losses could be assigned to each load. Theory of active and reactive loss allocation and branch-power-flow decomposition was proposed by Wu and Chen [46]. This has been used as the basis for allocating fixed costs and power losses under electricity deregulation. This paper is also based on proportional sharing and flow tracing in a network and uses a 4-bus network to demonstrate the obtained results. Energy Transaction Allocation Factors were introduced by Fradi, Brignone and Wollenberg [47]. The authors stated that MWs flows must be allocated to each line or group of lines in proportion to the MWs being transmitted by each transaction. A methodology was presented to calculate energy transaction allocation factors using process of integration of a first derivative function.

The allocation of transmission loss is dependent on market structure. A market could operate solely on the power pool system or combination of power pool and bilateral contracts. Different models for transmission loss allocation are possible within a certain market structure. These models can be developed on different criteria. One of the criteria is the supply of power due to transmission loss. The transmission loss could be supplied by the generating unit involved in contract with a customer or by any other generating unit which is located nearby the contracted load or it can be arranged by the ISO itself.

1.5.1 DEMAND AND LOSS PROVIDED BY THE SAME GENERATOR

In the M.Sc. work by the author [48], two methods have been developed to determine a generator's share of transmission loss in a fully deregulated power system. In the first method, a modified load flow is utilized to assess transmission losses and the method is termed as Incremental Load Flow Approach (ILFA). In the ILFA, a load flow solution is obtained by incrementing a customer load while the other loads are held at their previous levels. The resulting differential transmission loss is added to the transmission loss of the generator whose customer load has been incremented. The loads are incremented in an alternate sequence, in discrete steps, from zero to their respective levels.

In the other method, a mathematical model has been developed to determine the transmission loss that each of the sources is responsible for and the method is termed as Marginal Transmission Loss Approach (MTLA). The mathematical approach is based on Kron's [49] transmission loss expression and results in an iterative process. In a conventional system, total transmission loss is calculated on the basis of real and reactive power injections at the buses. This makes transmission loss expression a function of generations. But in a fully deregulated system, transmission loss expression is required to be a function of loads instead of generations. Transmission loss expression should reflect the effect of bilateral contracts in a network. By utilizing the MTLA, the transmission loss for each contract in a network can be separated. In the MTLA, a generator's share of the transmission loss can be found by making an incremental change in the generator's active power demand, while keeping all other loads fixed.

1.5.2 LOSS PROVIDED BY DIFFERENT GENERATORS/ISO

Every transaction of electricity involves some loss. Loss would increase with an increase in transportation distance. A customer may, however, want to buy power from a remote generator and this decision may be taken because of various considerations such as encouraging green power and subsidizing remote generating units. But the contracted generating unit may want to supply the load demand only due to various limitations e.g. capacity of the plant etc. In this situation the generating plant will depend on some other generating plants for the supply of transmission loss associated with the contract. The contracted plant might want a contract with another plant for supplying the loss occurred due to its contract with the customer. This would be a trilateral contract.

In another model, the generating utilities and the customers sign bilateral contracts and the ISO will be responsible for supplying the loss occurred due to all contracts. The ISO will determine a suitable plant based on location and price of electricity for the loss for the system. All bilateral parties involved would pay the ISO for their respective allocated losses.

1.5.3 CONTRACTED GENERATOR AS A PART OF ECONOMIC LOAD DISPATCH

Loads in a network follow some patterns and go high and low at different times of a day. Load forecast predicts the nature of load demands from load patterns and coming events with a high accuracy. The utilities use this prediction for determining the number of generating units required to meet the anticipated load, an essential activity of a power system, generally known as unit commitment.

Unit commitment dictates the number of generating units to be in spinning condition to meet load demands during the 24 hours. It also states the order of the units to be engaged in production according to the production cost of the units and starting time of the units. Production cost varies from unit to unit depending on their working principle. The production costs of hydro units are far less than those of the gas turbine and thermal units. It costs much more to produce energy in a gas turbine unit compared to that of a similar size thermal unit. The starting time of gas turbine and hydro units are much lower than conventional thermal plants. These two factors, production cost and starting time, determine the allocation of load among various units. This activity is commonly known as economic load dispatch.

A generator, in bilateral contract with a customer, is supposed to supply the required load. The contracted generator, however, may want to include itself in the economic load dispatch schedule of the power system network that it uses to supply the contracted load. When a generator becomes a part of the economic load dispatch schedule of the network, the term bilateral contract apparently does not exist anymore. But from the customer's point of view, there is nothing to object as long as its load demand is satisfied. From the Generator's point of view, it would want to be a part of the whole network, if it proves to be profitable. The producer would pay the utility/ISO for the increase in total cost due to its inclusion in the system.

1.5.4 COUNTER-FLOW

A system's transmission loss usually goes up when a generator is brought into the system to supply a load. The introduction of a generating unit however, may actually decrease the overall transmission loss depending upon its location in the network. This flow which opposes the initial flow in a particular transmission line is sometimes termed as counter-flow. Logically, a generator can contribute to decrease transmission loss only when there exists an initial flow in the opposite direction by another generator. Without the initial existing generator there would be no counter-flow in the system.

According to Gerge Gross and Shu Tao [50], 'Some transactions cause flow in the same directions as the net flow, while others cause flow in the opposite direction. The flow in the same direction as the net flow is called a dominant flow, while the flow in the opposite direction is a counter flow. Dominant flows increase the total transmission loss as the amount of the corresponding transaction is increased. Absent the dominant flow, the counter flow cannot exist. If the dominant flow disappears, the counter flow itself becomes dominant flow'.

Counter-flow is considered to be an important cost saving feature to a power system utility although it is a virtual term in power system analysis. It considers only the relative magnitudes of flow contributions in a line by the generators in a system. The concept of counter-flow stems from the relative position of suppliers (generating utilities) and buyers (loads) and their timing of entering the market with respect to each other. This relative position and timing make a difference in the overall transmission loss allocation.

It has been found that the determination of counter-flow depends on the sequence in which the loads or generators are brought into a system [50]. For example, if there are three loads, A, B and C, then for computational purposes the sequences in which the loads can be brought into the system are ABC, ACB, BAC, BCA, CAB and CBA [51,52]. If there is no counter-flow, then all six sequences would result in the same transmission loss. If there is a counter-flow then the transmission losses resulting from those sequences will differ from each other. The counter-flow in the system will also be dependent on the generators and loads' relative position in the system.

In a monopoly system, the effect of counter-flow has been considered only for transmission loss reduction. But, under deregulated system, counter-flow is of immense importance in transmission loss allocation among the IPPs. The allocation of transmission loss can be changed abruptly depending on the sequence used to determine it. Since counter-flow would increase the allocated loss of some generators and decrease the allocated loss of other generators, the choice of a particular sequence could become a subject of controversy.

1.6 ALLOCATION AND COSTING OF REACTIVE POWER

Power system networks consist of real and reactive elements. Almost in all cases, load is made up of real and reactive components. A system should not only be capable of meeting its real power demand but also should meet its reactive power demand.

Reactive power in a system can be controlled by adjusting the excitation of generating units and also by the use of reactive VAR compensators.

Any flow in a transmission line, real or reactive power, produces losses. The reactive transmission loss (I^2X) of the system is usually much higher that the real transmission (I^2R) loss, since the system reactance is generally several times the order of resistance.

The reactive power flow in a power system is a major source of system voltage drop. Any IPP which is in a bilateral contract with a customer, therefore, cannot intend to be in the business of supplying real power only, but be prepared to provide enough reactive power to maintain the system voltage to acceptable voltage levels in the system, hence ensure security and reliability.

If generators cannot provide sufficient reactive power due to operating constraints, the utility uses alternative methods to meet the reactive power demand of the system. There are various methods used to supply the needed reactive power in a power system e.g., synchronous condensers, over excited generators, static capacitors and static VAR.

1.6.1 ALLOCATION OF REACTIVE POWER

Any generator, which is willing to enter into a bilateral contract with a consumer, would be required to provide the real and reactive power demand of the customer. The power flow associated with a bilateral contract like any other power flow would cause transmission losses, both real and reactive. The allocation of reactive power loss associated with a bilateral contract can be easily calculated using one of the methods

[48] developed along with the allocation of the real power loss. In practice, a contracted generator may not be able to provide the required reactive load and the allocated reactive loss either due to its own limited capacity or system constraints. In situation like these a contracted generator will depend on other generators or the ISO for the supply of required reactive power at a cost premium.

1.6.2 REACTIVE POWER MARKET

In a power system network reactive power is an essential part of it. Reactive power must be provided to maintain the system bus voltage profile throughout the grid network. Deregulation process in power industry is creating a market for reactive power. Previously, some utilities charge power factor penalties to their customers as an indirect method for recovering the reactive power cost [53]. Researchers have shown that this practice is not capable of providing valid basis for the pricing of reactive power and it gives wrong price signals to customers, specially in a deregulated power system network. Li and David [54] have used the marginal cost at different buses to figure out the wheeling rates of reactive power in a system. Lamont and Fu [55] have reported a different way for the calculation of reactive power price in a deregulated environment. They used the most commonly used capital cost of generators which is normally provided in terms real power P in \$/MW. They converted this cost in terms of reactive power as follows:

$$\$ / MVA = \$ / (MW*pf)$$

$$\$ / MVAR = \$ / (MVA*\sin\theta) = \$ / (MVA*\sin(\cos^{-1}(pf))$$

Where,

pf = power factor of the generator

θ = power factor angle of the generator

Bhattacharya and Zhong [56] addressed the procurement challenge faced by an Independent System Operator (ISO) in deregulated power markets. A bidding structure is proposed considering generator's expectation of financial compensation. The paper suggested a two-tier approach to determine the most favorable contracts for reactive power supply for an ISO. Monte Carlo simulation is used in order to incorporate the uncertainty in reactive power demand. The proposed approach in the paper includes various constraints to have an optimal reactive power procurement structure for an ISO.

The above-mentioned methods considered reactive power as equivalent to real power for the sake of electricity spot pricing. But in practice, the concept of reactive power is far more complicated and hence, has to be handled differently. ISO needs reactive power in order to maintain sound operation of the grid network. In a pool system, like California, the ISO determines the reactive power support requirement using load flow study on a daily basis [57]. A generator in a bilateral contract is responsible for the supply of reactive power required to meet the contracted load demand. Due to system constraints a generator in contract may not be able to produce sufficient reactive power. In this situation it will depend on other generators or IPPs for the fulfillment of reactive power demand. Other generators or IPPs may take this opportunity to sell reactive power. However, in order to settle for a price for the reactive power an IPP should know production cost of the reactive power. Unlike real power, production cost of reactive power does not depend on fuel cost if the generator is already spinning. Reactive power production involves variable excitation of a generator. Hence, the related cost of reactive power is mainly due to the variation in excitation of the machine and in many cases could be very small. An IPP may decide whether to sell reactive power or stick to sell of real power only once it knows the production cost of commodities and their market prices.

1.7 SPINNING RESERVE IN A DEREGULATED SYSTEM

In a power system network, the total installed capacity is usually higher than the peak system load. The required generating capacity in a power system depends on the availability of generating units and the load pattern. Any healthy power system must have additional generating capacity. This additional capacity of the generating units is required for unplanned or forced outages as well as normal maintenance of units. The additional capacity of the generators that is spinning that can take additional load is known as spinning reserve.

The spinning reserve provides the ability to increase generation output immediately and automatically in response to any increased load demand or system contingencies. The reserve must be capable of providing power to meet any unforeseen changes in the total system load. Some services like medical facilities, computer-based operations, communication area etc. require continuous power supply. When continuity of operation is required, supply redundancy must be carried throughout the system.

In a conventional power system network, two basic concepts are used to calculate the required amount of spinning reserve [58]

- Deterministic Approach
- Probabilistic Approach

Deterministic Approach – this approach used fixed amount of generation for spinning reserve and does not take the probability of failures (of generators, transmission line etc.) into account. The spinning reserve can found using one of the following methods

- Percentage of system load or operating capacity
- Fixed capacity margin
- Largest contingency
- Any combination of the above methods

There is a wide variation in terms of the methods used by the utilities in North America for spinning reserve calculation. But the utilities using deterministic approach have one thing in common, they do not consider the probability of component failure in their calculation.

Probabilistic Approach – This approach basically uses the stochastic nature of system components in its assessment of the required spinning reserve.

Although much of the research works on deregulated power system are focused on transmission access and losses, little has been published on spinning reserve. Very few works have been reported on this. Tseng et al. [59] have formulated a price-based adaptive spinning reserve requirements. The authors have applied Lagrangian relaxation method using game theory and oligopoly to solve the unit commitment. In deregulated electricity market, spinning reserve is obtained and priced as an ancillary service. The

ISO has the responsibility for meeting the reserve requirements based on predetermined operating guidelines. The authors proposed a scheme for obtaining an optimal level of spinning reserve at or above a minimum requirement. They have shown that the optimum solution of the Lagrangian relaxation method for unit commitment is actually a Nash equilibrium. In another paper, Arroyo and Conejo [60] proposed a method that helps a power generator to optimally determine its involvement in the power pool and spinning reserve market. They considered Ten-Minute and Thirty-Minute reserve markets along with regular energy market. Based on the forecasted price their model helps an IPP to decide the production level of real power and spinning reserve in order to maximize its profit.

The responsibility for carrying spinning reserve under a deregulated environment becomes a complex issue too. It is difficult to figure out whose responsibility is it to carry the reserve and what should be paid for carrying it. In present, some systems like in California, ISO is responsible for providing the spinning reserve [27]. But without any monetary interest for the ISO the situation becomes more critical than ideally it is thought to be.

The two possible scenarios for deregulation can be considered again - pool system and bilateral contracts. In a pool system an ISO is responsible for spinning reserve. There would be a number of IPPs in the market place competing each other for selling energy to the ISO. Some IPPs would try to sell real-time power to the ISO and some would try to bid for their spinning reserve. They would earn money for providing the spinning reserve. The decision of the IPPs, whether to sell real-time power or just spin their generators, would depend on the return of their investment in this sector and expected price of these commodities. A higher price for spinning reserve would drive most of the real-time power provider towards the bidding for spinning reserve and vice versa. But ramp-up time of generators restricts them from bidding their full capacity at the spinning reserve market. There must be a balance in the demand and price of energy and number of IPPs acting as real-time power provider and spinning reserve provider. Ultimately the market would take control and force all IPPS to an equilibrium state. All IPPs will act in such a way so that their profits are maximized. In the long run equilibrium profit of each IPP will become zero. A model has been developed that would be useful for the IPPs to decide their mode of operation.

In a bilateral agreement, an IPP is responsible for providing power to its contracted customer. The contract could be made in different ways. The customer may want to buy energy with a high degree of reliability by paying more money or the customer may opt for a lesser degree of reliability by paying less. The IPP would have to ensure that the energy is supplied with the pre-specified level of reliability. In order to maintain a pre-specified level of reliability an IPP may maintain a certain magnitude of spinning reserve or may buy emergency power from another IPP. An IPP may want its ISO to maintain the required spinning reserve. The ISO, in this case, would charge the IPP for spinning reserve as well as for the network access.

In market economy, the price of any product is set by its demand and supply. Under fully deregulated environment, the energy sector would be dominated by the same principle. In some extreme cases, there might be no existence of spinning reserve, if spinning a generator proves to be less profitable than selling real-time power. The customers, who are willing to pay for highly reliable energy, would get a higher priority as they would be paying more. On the other hand, service to the customers, who are not willing to pay more or who can endure power outages, might be interrupted more frequently. The priority would be set on the basis of customers' willingness to pay which is the principle of market economy.

1.8 OBJECTIVES AND SCOPE OF THE RESEARCH

In light of the changing perspective of the power system networks and their operation, the followings are investigated as a part of this Ph.D. research.

a) **Development of a generalized transmission loss allocation model** – it is desired to develop a generalized transmission loss allocation model which can be utilized to assess loss allocations in a bilateral contract between a generator and a customer in a deregulated power system network.

An energy producing agent or an IPP may explore various options to fulfill its obligation with respect to transmission loss. The following options would be explored.

i) Demand and Loss provided by the same generator – a generator may want to supply its contracted customer load and allocated transmission loss.

ii) Loss provided by different generators/ISO – a contracted generator would supply the customer load and buy the required loss from another generator in the system.

iii) Contracted generator as a part of economic load dispatch – a contracted generator may want to operate within the economic load operation to supply its customer load.

iv) Counter-flow – the existence of counter-flow can be determined by changing the sequence of the loads or generators during the calculation of transmission loss. However, a change in the sequence would result a change in the transmission loss allocation. A suitable method would be developed by the candidate to allocation of transmission losses in the presence of counter-flows.

b) **Reactive power market** – In a bilateral contract, a generator is supposed to provide customer's real and reactive power demand. Due to system constraints it may not be able to supply the reactive demand of the contracted customer. In this case, the contracted generator would buy reactive power from another generator. Hence it is important to find the basis of minimum asking price of the reactive power that a generator would sell to the market or another generator. This is an optimization problem for a single IPP which is interested to sell both real and reactive power in market place. A model is intended to develop to solve this optimization problem of an IPP.

c) **Spinning Reserve Market** – An IPP may be interested in selling spinning reserve as well as real and reactive power. An optimization technique is required to help an IPP to obtain maximum profit from selling the three commodities. A suitable model would be developed to assist an individual power supplier to solve this optimization problem.

1.9 OUTLINE OF THE THESIS

The thesis is organized in seven chapters. Chapters 1 and 2 deal with the background of deregulation, existing power systems operation in different countries, market structure and basic concepts of power systems. Power systems in different countries have adopted different methods for operation and the on-going wave of deregulation is changing the previous setup. Political situation, environmental pressures, technological advancements, economical conditions etc. are the significant factors in making decision for bringing any change in power system operation. Different countries have different challenges to face and hence operations of power system vary greatly from country to country. With the implementation of deregulation, energy market scenario is changing. Market structure is changing too. These issues are briefly discussed in Chapter 1. Chapter 2 describes transmission losses and its role in power system operation. The effect of transmission losses is discussed for monopoly and pool operation and bilateral contracts. AC load flow analysis is described briefly to show how to calculate transmission losses in power system networks. This chapter also provides the mathematical basis for the Marginal Transmission Loss Approach.

AC load flow analysis has been utilized to develop one of the proposed methods for transmission loss allocation. A test system is described which has been used for analysis in the following chapters. In addition to the test system, the IEEE Reliability Test System is also used to test and validate the developed methods. Details of the IEEE Reliability Test System are shown in Appendix-A. The Incremental Load Flow Approach (ILFA) is presented in Chapter 3 along with numerical examples. Various load conditions have been considered for the test system and the transmission loss allocations have been shown in tables as well as plotted in graphs. The test system is assumed to operate on bilateral contracts only. The IEEE Reliability Test System is used to show how the developed methods can be applied to assess transmission loss allocation in a hybrid system of power pool and bilateral contracts.

The loss allocations obtained by the ILFA have been verified by the Marginal Transmission Loss Approach (MTLA) in Chapter 4. The mathematical formulation of the MTLA is reported in this chapter. It involves complex mathematical and matrix manipulations resulting in a set of quadratic equations. This set of quadratic equations can be used to obtain transmission loss allocation for any bilateral contract between a generating entity and a customer in a power system network. Load conditions similar to those discussed in Chapter 3 have been applied for both the test system and the IEEE Reliability Test System to obtain loss allocations. The loss allocations resulting from the two methods are compared. This chapter includes graphs of loss allocations for various load conditions to provide a better view of the methods.

In a deregulated environment, a generating entity may have different options for supplying power to its contracted customer load. It may supply from its own resources or may become part of an economic load dispatch. A brief description of power system operation including economic load dispatch is covered in Chapter 5. Cost analysis of different available choices for a generating plant is shown in this chapter. Counter-flow is one of the most contentious issues in power system operation especially in a deregulated environment. Different examples are shown to indicate the existence of counter-flow in some power system network in Chapter 5. This chapter also shows that the developed methods for transmission loss allocation, the ILFA and the MTLA, are capable of taking care of counter-flow in a power system network.

Reactive power supply and its optimal productions are discussed in Chapter 6. A generator locked in a bilateral contract may not be able or even allowed to provide its customer with required demand of reactive power supply. Different technical reasons may cause it but this situation makes a generating entity dependent on others. Chapter 6 includes a suitable solution to a minimum acceptable price of reactive power considering opportunity cost for the supplying generator. A composite supply curve for reactive power from multiple generators of different ratings is also shown in this chapter. This chapter also includes profit maximization model for an IPP based on the forecasted or expected price of three possible commodities in a power market– real power, reactive power and spinning reserve. The model would help an IPP to decide the production levels of these three commodities to realize maximum profit. Zero profit conditions are considered along with the profit maximization model to determine the minimum acceptable price vectors of these three commodities for an IPP. Finally Chapter 7 reports conclusions brief overview of the work reported in this thesis.

CHAPTER 2: TRANSMISSION LOSS IN AN ELECTRIC POWER SYSTEM

2.1 Transmission Loss

A Transmission network is an electrical highway through which electrical energy flows. It connects consumers to the generators or producers of electrical power. Since the transmission lines are made of physical conductors, loss of electrical power occurs in the line during its flow and it is quite significant in a large network. Transmission losses are unavoidable, and therefore, generating units are supposed to supply the losses along with the loads in a network.

Transmission loss can be divided into two parts: real and reactive. Real power generated by a system should be equal to the sum of all its loads and line losses. It is needless to say that without providing the transmission loss, the system load could not be supplied.

On the other hand, reactive power in a system is required for system voltage stability. Reactive power loss must be provided for maintaining the required voltage profile in a network. Unlike real power, reactive power does not have any direct monetary value but it is an extremely important safety factor in power system security and satisfactory operation.

Total transmission loss in a system can be calculated in different ways. One of the popular ways is to utilize Kron's [49] transmission loss formula. This formula is based on a number of assumptions and calculates transmission loss in terms of generations. In spite of the assumptions, Kron's formula gives a fairly close result when compared to the results obtained through more accurate methods. The main advantage of this formula is that it is a function of generation only, and therefore, can be applied to find approximate transmission loss in a system by knowing the generation of individual plants of the system.

Transmission loss can also be calculated using standard AC load flow analysis. AC load flow analysis is used widely to obtain line flows, bus angles, contingency enumeration, bus voltages etc. for the smooth operation of power system network.

Deregulation in the electric power industry has brought several changes to the way electricity producers operate and do business. Deregulation allows producers to bid for the sale of their energy and also allows customers to bid for the purchase of this energy. In most jurisdictions the bidding process and the resulting energy exchanges take place in the form of a power pool operation and an independent system operator (ISO) manages and controls this operation of the power pool. One of the important tasks of an ISO is to ensure the balance of supply and demand on a real time basis. An ISO fulfills this objective by buying energy on the spot market and it also buys extra energy from a predetermined supplier or from the spot market to make up for transmission losses. However, one of the most important aspects of deregulation is that it allows bilateral contracts between producers and customers.

In a bilateral contract the supplier usually produces enough power to meet its contracted load and the resulting transmission loss. In the presence of multiple bilateral contracts in a system, the ISO has to allocate the transmission loss to the appropriate parties in a suitable manner. It is very important to determine the proper allocation of transmission network loss and the responsible party for supplying this loss in a deregulated power system network.

2.1.1 Transmission Loss in a Conventional Monopoly System

In a conventional system, a utility or company has the absolute right to market electricity in a designated service area. For Example, in Saskatchewan, SaskPower produces power, transmits it using the transmission network and delivers it to the residents of Saskatchewan. City corporations in Saskatoon and Regina, act as sole distributors beside SaskPower.

In a monopoly system, transmission loss is the full liability of the utility that has the sole responsibility of producing and supplying energy in its jurisdiction. An electric power utility operates the system under its jurisdiction to minimize the overall operating cost. This does not necessarily mean transmission loss minimization. A utility may use low cost generators in order to minimize its total operating cost, but its transmission loss may not be optimized. In a monopoly system, a utility determines which of its generator should provide the required transmission loss in the network for an optimal economic operation.

2.1.2 Transmission Loss in Pool System

A power pool is the most common form of market at present due to its simple nature. The generating utilities or IPP and customers both bid for buying and selling power at the power pool. A power pool conducts different types of auctions: day ahead market, hour ahead market, real time spot-pricing market etc. The generating utilities (IPPs) in a deregulated environment can operate under a pool system or bilateral contracts. Under the pool system, the ISO in charge of the pool will post the hourly demand and possible price structure and the IPP will determine its optimal operating conditions based on these facts. The other factors like size and location of the generating plant, minimum power production requirement would also have effect on the operation of the IPPs.

In a power pool system, the generating utilities or the IPPs only sell power to the pool and the ISO administers the system as well as the transmission loss in the system. The ISO calculates transmission loss and incorporates the cost of supplying this transmission loss in the full price that it charges to the customer.

2.1.3 Transmission Loss in a System with Bilateral Contracts

In a deregulated power system, any electric power generating utility or independent power producer (IPP) would be allowed to enter into a bilateral contract with any customer. Generating utilities and customers contract each other for selling and buying of power. The seller with the help of the system operator arranges the transportation of the contratced power over the transmission network.

The transmission loss in a bilateral contract can be handled in two ways; with the direct participation of the ISO or, without the direct participation of the ISO. In a direct participation of the ISO, the contracting parties would let the ISO supply the transmission loss from a producer of its choice and the parties would share the resulting cost according to an agreed upon formula. In an indirect participation of the ISO, the contracting producers would produce enough power to meet its contractual obligation as well as the resulting transmission loss. The role of the ISO would be to simply transport the energy from the producer to the customer in exchange of an agreed upon wheeling fee. An indirect participation of an ISO is assumed in the research work reported in this thesis.

2.2 Transmission Loss Calculation Using AC Load Flow

It has been stated before that transmission loss can be calculated in different ways. In a conventional monopoly system AC load flow analysis can be used to determine it. Kron's transmission loss formula is also used to determine the transmission loss in an electric power system network. Before explaining the AC load flow analysis a brief description of power system operation is given below.

2.2.1 Power System Operation

A Power system is a composite complex entity. Its three functional areas are generation, transmission and distribution. The combination of these three functions and their optimized utilization is the goal of any power system operation. The generation may consists of different kinds of power plants e.g. thermal, hydro and nuclear. An operating authority wants to use the available generating units in an optimized and efficient way. Loads in a network vary during the day and also during various seasons. Efficient power system utilities take all those factors into consideration and operate for minimal operating cost.

Loads in a network follow some patterns and go high and low at different times of a day. Load forecast predicts the nature of load demand from load patterns and coming events with a high accuracy. The utilities use this prediction for determining the number of generating units required to meet the anticipated load, an essential activity of a power system, generally known as unit commitment.

Unit commitment dictates the number of generating units to be in spinning condition to meet load demands during the 24 hours. It also states the order of the units to be engaged in production according to the production cost of the units and starting time of the units. Production cost varies from unit to unit depending on their age and working principle. The production cost of hydro units are far less that those of the gas turbines and thermal units. It costs much more to produce energy in a gas turbine unit compared to that of a similar size thermal unit. The starting times of gas turbines and hydro units are much lower than conventional thermal plants. These two factors, production cost and starting time, determine the allocation of load among various units. This activity is commonly known as Load Dispatch.

Once a load dispatching schedule is prepared, the feasibility of the schedule is checked with the help of an AC load flow analysis. An AC load flow analysis usually provides bus voltages, line flows and active reactive power mismatch.

2.2.2 AC Load Flow Technique

Load flow analyses are essential to the operation and study of a power system. In fact, load flow forms the heart of power system analysis. Load flow analysis plays a key role in the planning or expansions of transmission and generation facilities.

In general, a load flow analysis solves for unknown bus voltages and unspecified generation and finally for the complex power flow in the network components for a given power system network, with known loads and some set of specifications or restrictions on power generations and voltages. The losses in individual components and in the total network as a whole are usually calculated. The system is checked for component overloads and voltage outside allowable tolerance band.

Basically, two methods are widely used for load flow analyses. They are: Gauss-Seidal and Newton-Raphson methods. Both methods need certain input parameters for performing an analysis. The Newton-Raphson method is widely accepted in the power industry and is used in this work. Y-bus or Z-bus is required to be calculated before proceeding for a solution. Buses in a network are divided into three categories: swing bus, generator or PV bus and load or PQ bus. Each bus is associated with four parameters: voltage magnitude, phase angle and real and reactive powers.

Swing bus - it is a generator bus whose voltage and angle have been specified. The real and reactive power generations are to be calculated.

Generator bus - the bus voltage and real power generated have to be specified and the reactive power and the bus angle are to be determined.

Load bus - the bus where a load is connected and the real and reactive magnitude of the load are known. The bus voltage and angle have to be determined.

Either of the load flow techniques actually solves a set of simultaneous algebraic equations in an iterative way. Newton-Raphson method is widely used for load flow analysis for its inherent simplicity of fast convergence rate. The following simultaneous equations are required for the solution of a load flow by Newton-Raphson method. At Bus k,

$$P_k = \sum_{n=1}^{N} V_k V_n Y_{kn} \cos(\theta_{kn} + \delta_n - \delta_k) \tag{2.1}$$

$$Q_k = -\sum_{n=1}^{N} V_k V_n Y_{kn} \sin(\theta_{kn} + \delta_n - \delta_k) \tag{2.2}$$

Where,

P_k = real power at bus k

Q_k = reactive power at bus k

V_k = rms voltage at bus k

V_n = rms voltage at bus n

Y_{kn} = element of bus admittance matrix between buses k and n

θ_{kn} = angle associated with Y_{kn}

δ_k = bus angle of bus k in radians

The method starts with some initial values for the specified parameters, P and Q for every bus except the swing bus. Estimated values of V and δ for each bus except the swing bus, for which they are known, are used to calculate the same parameters. The mismatch in power calculation originating from specified and calculated values are determined for each bus.

$$\Delta P_k^{(0)} = P_{ks} - P_{kc}^{(0)}$$

$$\Delta Q_k^{(0)} = Q_{ks} - Q_{kc}^{(0)}$$

where the subscripts s and c mean specified and calculated values respectively and the superscript represents the iteration number. In the next step the Jacobian (J) is determined.

$$J = \begin{bmatrix} \frac{\partial P_1}{\partial \delta_1} & \cdots & \frac{\partial P_1}{\partial \delta_{N-1}} & \frac{\partial P_1}{\partial V_1} & \cdots & \frac{\partial P_1}{\partial V_{N-1}} \\ \cdots & & & & & \\ \frac{\partial P_{N-1}}{\partial \delta_1} & \cdots & \frac{\partial P_{N-1}}{\partial \delta_{N-1}} & \frac{\partial P_{N-1}}{\partial V_1} & \cdots & \frac{\partial P_{N-1}}{\partial V_{N-1}} \\ \frac{\partial Q_1}{\partial \delta_1} & \cdots & \frac{\partial Q_1}{\partial \delta_{N-1}} & \frac{\partial Q_1}{\partial V_1} & \cdots & \frac{\partial Q_1}{\partial V_{N-1}} \\ \cdots & & & & & \\ \frac{\partial Q_{N-1}}{\partial \delta_1} & \cdots & \frac{\partial Q_{N-1}}{\partial \delta_{N-1}} & \frac{\partial Q_{N-1}}{\partial V_1} & \cdots & \frac{\partial Q_{N-1}}{\partial V_{N-1}} \end{bmatrix}$$

$$\begin{bmatrix} \Delta P_1^{(0)} \\ \cdots \\ \Delta P_{N-1}^{(0)} \\ \Delta Q_1^{(0)} \\ \cdots \\ \Delta Q_{N-1}^{(0)} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_1}{\partial \delta_1} & \cdots & \frac{\partial P_1}{\partial \delta_{N-1}} & \frac{\partial P_1}{\partial V_1} & \cdots & \frac{\partial P_1}{\partial V_{N-1}} \\ \cdots & & & & & \\ \frac{\partial P_{N-1}}{\partial \delta_1} & \cdots & \frac{\partial P_{N-1}}{\partial \delta_{N-1}} & \frac{\partial P_{N-1}}{\partial V_1} & \cdots & \frac{\partial P_{N-1}}{\partial V_{N-1}} \\ \frac{\partial Q_1}{\partial \delta_1} & \cdots & \frac{\partial Q_1}{\partial \delta_{N-1}} & \frac{\partial Q_1}{\partial V_1} & \cdots & \frac{\partial Q_1}{\partial V_{N-1}} \\ \cdots & & & & & \\ \frac{\partial Q_{N-1}}{\partial \delta_1} & \cdots & \frac{\partial Q_{N-1}}{\partial \delta_{N-1}} & \frac{\partial Q_{N-1}}{\partial V_1} & \cdots & \frac{\partial Q_{N-1}}{\partial V_{N-1}} \end{bmatrix} \begin{bmatrix} \Delta \delta_1^{(0)} \\ \cdots \\ \Delta \delta_{N-1}^{(0)} \\ \Delta V_1^{(0)} \\ \cdots \\ \Delta V_{N-1}^{(0)} \end{bmatrix} \qquad (2.3)$$

Equation (2.3) can be written as

$$\begin{bmatrix} \Delta P^k \\ \Delta Q^k \end{bmatrix} = [J] \begin{bmatrix} \Delta \delta^k \\ \Delta V^k \end{bmatrix} \qquad \text{or}$$

$$\begin{bmatrix} \Delta \delta^k \\ \Delta V^k \end{bmatrix} = [J]^{-1} \begin{bmatrix} \Delta P^k \\ \Delta Q^k \end{bmatrix}$$ where (J) is the inverse of the Jacobian Matrix

Equation (2.3) is solved by inverting the Jacobian (J) and errors in voltages and angles are calculated. New values of V and δ are estimated by subtracting these errors from respective previous values. These new voltages and angles are then used to calculate new bus powers using Equations (2.1) and (2.2). This process is repeated until the mismatch at each bus comes down within a tolerable limit. The Kirchhoff's law, the algebraic sum of all flows at a bus must be zero, should be satisfied by any load flow solution and can be used as a convergence constraint.

2.2.3 Example of Load Flow Analysis

A small network is shown in Figure 2.1 to illustrate the load flow analysis. Bus 1, 2 and 3 are defined as swing bus, generator bus and load bus respectively. Line and generator parameters are shown in Tables 2.1 and 2.2. The base values are 100 MVA and 138 kV.

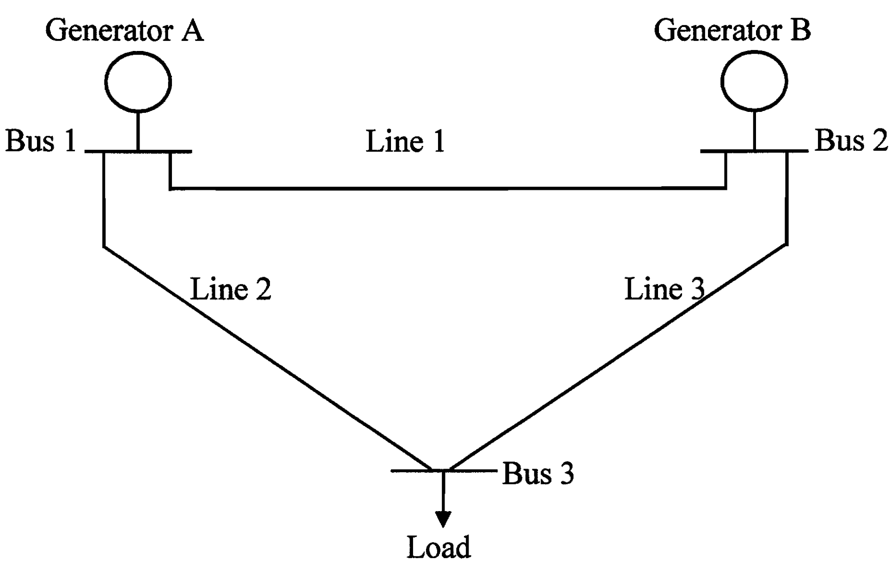

Fig.2.1 : A small power system network.

Table 2.1: Line parameters for the system shown in Figure 2.1.

Line Number	From Bus	To Bus	Length (km)	Resistance p.u.	Reactance p.u.
1	1	2	80	0.01575	0.07876
2	1	3	50	0.0105	0.05251
3	2	3	100	0.02625	0.13127

Table 2.2: Generating unit characteristics for the system shown in Figure 2.1.

Generating Unit	Cost Function ($/hr)	Maximum Output (MW)	Minimum Output (MW)
Generator A	$0.022P_1^2+12.45P_1+70$	500	90
Generator B	$0.024P_2^2+13.65P_2+80$	400	40

The real and reactive power demands are 200 MW and 80 MVAR respectively and supplied by both units.

The simultaneous equations for the system shown in Figure 2.1 are

$$P_2 = V_2V_1Y_{21}\cos(\theta_{21}+\delta_1-\delta_2)+V_2^2Y_{22}\cos(\theta_{22})+V_2V_3Y_{23}\cos(\theta_{23}+\delta_3-\delta_2) \quad (2.4)$$

$$P_3 = V_3V_1Y_{31}\cos(\theta_{31}+\delta_1-\delta_3)+V_3V_2Y_{32}\cos(\theta_{32}+\delta_2-\delta_3)+V_3^2Y_{33}\cos(\theta_{33}) \quad (2.5)$$

$$Q_2 = -V_2V_1Y_{21}\sin(\theta_{21}+\delta_1-\delta_2)-V_2^2Y_{22}\sin(\theta_{22})-V_2V_3Y_{23}\sin(\theta_{23}+\delta_3-\delta_2) \quad (2.6)$$

$$Q_3 = -V_3V_1Y_{31}\sin(\theta_{31}+\delta_1-\delta_3)-V_3V_2Y_{32}\sin(\theta_{32}+\delta_2-\delta_3)-V_3^2Y_{33}\sin(\theta_{33}) \quad (2.7)$$

Since Bus 2 is a generator bus and voltage V_2 for this bus has been specified, Equations (2.4), (2.5) and (2.7) are sufficient for obtaining the solutions of the load flow analysis.

All unspecified voltages and angles are initially estimated as 1 p.u. and zero radian initially.

Equation (2.3) for the current system stands as:

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_3 \end{bmatrix} = \begin{bmatrix} 19.53 & -7.33 & -1.46 \\ -7.33 & 25.63 & 5.12 \\ 1.46 & -5.12 & 25.65 \end{bmatrix}^{-1} \begin{bmatrix} 0.798 \\ 1.998 \\ 0.7902 \end{bmatrix}$$

The errors in initial estimated voltages and angles after the first iteration are

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_3 \end{bmatrix} = \begin{bmatrix} 0.0785 \\ 0.0915 \\ 0.0446 \end{bmatrix}$$

The errors in voltages and angles are subtracted from their initial estimates to get a new set of estimated values. The new estimated values are:

$$\begin{bmatrix} \delta_2 \\ \delta_3 \\ V_3 \end{bmatrix} = \begin{bmatrix} -0.0785 \\ -0.0915 \\ 0.9554 \end{bmatrix}$$

In the next iteration Equation (2.3) stands as:

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_3 \end{bmatrix} = \begin{bmatrix} 19.32 & -6.86 & -1.94 \\ -7.08 & 24.09 & 6.57 \\ 0.89 & -2.91 & 23.74 \end{bmatrix}^{-1} \begin{bmatrix} 0.022 \\ -0.103 \\ -0.094 \end{bmatrix}$$

The error matrix becomes:

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_3 \end{bmatrix} = \begin{bmatrix} -0.0004 \\ -0.0032 \\ -0.0043 \end{bmatrix}$$

The new estimated values are:

$$\begin{bmatrix} \delta_2 \\ \delta_3 \\ V_3 \end{bmatrix} = \begin{bmatrix} -0.0781 \\ -0.0883 \\ 0.9511 \end{bmatrix}$$

This process continues unless the errors come down within a pre-specified tolerable limit. The solutions of the AC load flow analysis are shown in Tables 2.3 and 2.4.

Table 2.3: AC load flow solution for the system shown in Figure 2.1.

Bus Type	Bus	Voltage p.u.	Phase Angle degrees	Real Generation p.u.	Reactive Generation p.u.	Real Load p.u.	Reactive Load p.u.
Swing	1	1.0	0.0	1.2391	0.7493	0.0	0.0
Generator	2	1.0	0.7222	0.8	0.2463	0.0	0.0
Load	3	0.95	-3.9598	0.0	0.0	2.0	0.8

Table 2.4: Line flows in the network shown in Figure 2.1.

Line	From Bus	To Bus	Real Power p.u.	Reactive Power p.u.
1	1	2	-0.1537	0.0317
	2	1	0.1541	-0.0298
2	1	3	1.3928	0.7175
	3	1	-1.367	-0.5886
3	2	3	0.6459	0.2761
	3	2	-0.633	-0.2114

The system shown in Figure 2.1 is small and Tables 2.3 and 2.4 show the typical format of load flow outputs which include bus voltages, angles, generation and line flows. Generator A connected to the swing bus is supplying (1.2391 + j0.7493 p.u.), a major portion of the demand, while generator B is providing 0.8 + j0.2463 p.u. to meet the load. The voltage at Bus 3 is 0.95 p.u. because of the load connected to it. This voltage can be enhanced by connecting a tap-changing transformer to this bus. Table 2.4 shows the line flows. It is found from Table 2.4 that line 2 is carrying the maximum power (1.3928 + j0.7175 p.u.) because of the system configuration. Although Buses 1 and 2 have the same voltage magnitude, a small amount of power flows from bus 2 to bus 1 due to a difference between their bus angles.

2.2.4 Transmission Loss Calculation from Load Flow

A load flow analysis can be utilized to determine total transmission loss in a system as well as losses in individual components i.e., in transformers or transmission lines. A load flow analysis provides real and reactive powers at different buses. Total transmission loss can be calculated easily from the algebraic sum of powers at all buses. Loss in an individual line can be determined by power calculation at two buses that are connected by the particular line. The difference between the line flows at the two ends of a single line indicates the transmission loss in that line. For example, 1.3928 p.u. real power flows from Bus 1 to 3 and 1.367 p.u. real power flows from bus 3 to 1. The difference between these two flows, 0.0258 p.u., represents the line loss in line 1. Total real and reactive line losses are 0.0391 p.u. and 0.1956 p.u. respectively for the system shown in Figure 2.1.

The loss calculation by load flow is more accurate than any other method.

2.3 Transmission Loss Calculation

Kron published his very famous transmission loss formula in 1952 based on general transmission loss formula [49]. It is an approximate transmission loss formula as a function of generations in a power system network. This loss formula is simple and calculates transmission loss directly without any iteration. General transmission loss formula has been used as the basis for the generalized Marginal Transmission Loss Approach reported in this thesis.

2.3.1 Transmission Loss Expression

Mathematically, transmission loss is the sum of complex powers injected at all buses in a network.

$$S_l = P_l + jQ_l = \sum_i S_i \qquad (2.8a)$$

or,

$$S_i = V_i I_i^*$$

where,

S_l = total complex power loss

P_l = total real power loss

Q_l = total reactive power loss

S_i = complex power at bus i

V_i = voltage at bus i and

I^*_i = complex conjugate of bus current at bus i.

Equation (2.8a) can be written as

$$S_l = [V_B]^T [I_B]^* \qquad (2.8b)$$

where,

$$\begin{aligned}
[V_B] &= [Z_B]\,[I_B] \\
[Z_B] &= [R] + [jX] \\
[I_B] &= [I_p] + [jI_q]
\end{aligned}$$

$[Z_B]$ = bus impedance matrix of the system

$[R]$, $[X]$ = real and reactive components of the bus impedance matrix

$[I_B]$ = bus current matrix

$[I_p]$, $[I_q]$= real and reactive components of the bus current matrix

Replacing V_B and I_B by their real and imaginary parts, Equation (2.8b) becomes:

$$\begin{aligned}
S_l &= [I_B]^T [Z_B]^T [I_B]^* \\
&= \left([I_p] + [jI_q]\right)^T \left([R] + [jX]\right)\left([I_p] + [jI_q]\right)^*
\end{aligned}$$

After separating the real and imaginary parts

$$P_l = [I_p]^T[R][I_p] + [I_q]^T[R][I_q] \tag{2.9}$$

and

$$Q_l = [I_p]^T[X][I_p] + [I_q]^T[X][I_q] \tag{2.10}$$

At any bus i the following relationship holds

$$P_i + jQ_i = V_i I_i^* \tag{2.11}$$

where,

$$V_i = V_i\left(\cos\delta_i + j\sin\delta_i\right)$$

$$I_i = I_{pi} + jI_{qi}$$

V_i = voltage at bus i

I_i = bus current at bus i

Equation (2.11) can be written as

$$P_i + jQ_i = V_i(\cos\delta_i + j\sin\delta_i)(I_{pi} + jI_{qi})^* \tag{2.12}$$

Equating the real and imaginary parts of Equation (2.12)

$$P_i = V_i I_{pi}\cos\delta_i + V_i I_{qi}\sin\delta_i \tag{2.13}$$

$$Q_i = V_i I_{pi}\sin\delta_i - V_i I_{qi}\cos\delta_i \tag{2.14}$$

Solving Equations (2.13) and (2.14) for I_{pi} and I_{qi}

$$I_{pi} = \frac{P_i\cos\delta_i + Q_i\sin\delta_i}{V_i}$$

$$I_{qi} = \frac{P_i\sin\delta_i - Q_i\cos\delta_i}{V_i}$$

or in vector form

$$\left[I_p\right] = [C][P] + [D][Q] \tag{2.15}$$

$$\left[I_q\right] = [D][P] - [C][Q] \tag{2.16}$$

where,

$[C]$ = diagonal matrix with elements $\left(\cos\delta_i \big/ V_i \right)$

$[D]$ = diagonal matrix with elements $\left(\sin\delta_i \big/ V_i \right)$

Now Equation (2.9), for the real part of the transmission loss, becomes

$$\begin{aligned} P_l &= \left[I_p\right]^T [R]\left[I_p\right] + \left[I_q\right]^T [R]\left[I_q\right] \\ &= \left([C][P] + [D][Q]\right)^T [R]\left([C][P] + [D][Q]\right) + \left([D][P] - [C][Q]\right)^T [R]\left([D][P] + [C][Q]\right) \end{aligned}$$

or,

$$\begin{aligned} P_l = {} & [P]^T \left([C]^T [R][C] + [D]^T [R][D]\right)[P] - [P]^T \left([D]^T [R][C] + [C]^T [R][D]\right)[Q] \\ & + [Q]^T \left([D]^T [R][C] - [C]^T [R][D]\right)[P] + [Q]^T \left([C]^T [R][C] + [D]^T [R][D]\right)[Q] \end{aligned}$$

which can be written in matrix form as:

$$P_l = \begin{bmatrix} [P]^T & [Q]^T \end{bmatrix} \begin{bmatrix} [A_p] & -[B_p] \\ [B_p] & [A_p] \end{bmatrix} \begin{bmatrix} [P] \\ [Q] \end{bmatrix} \tag{2.17}$$

where,

$$\begin{aligned} \left[A_p\right] &= [C]^T [R][C] + [D]^T [R][D] \\ \left[B_p\right] &= [D]^T [R][C] - [C]^T [R][D] \end{aligned}$$

and

$$[C]^T[R][C]=\begin{bmatrix} c_1 & 0 & 0\ldots & 0 \\ 0 & c_2 & 0\ldots & 0 \\ \ldots & & & \\ 0 & 0 & 0\ldots & c_n \end{bmatrix}\begin{bmatrix} r_{11} & r_{12} & \ldots & r_{1n} \\ r_{21} & r_{22} & \ldots & r_{2n} \\ \ldots & & & \\ r_{n1} & r_{n2} & \ldots & r_{nn} \end{bmatrix}\begin{bmatrix} c_1 & 0 & 0\ldots & 0 \\ 0 & c_2 & 0\ldots & 0 \\ \ldots & & & \\ 0 & 0 & 0\ldots & c_n \end{bmatrix}$$

$$=\begin{bmatrix} c_1 r_{11} c_1 & c_1 r_{12} c_2 & \ldots & c_1 r_{1n} c_n \\ c_2 r_{21} c_1 & c_2 r_{22} c_2 & \ldots & c_2 r_{2n} c_n \\ \ldots & & & \\ c_n r_{n1} c_1 & c_n r_{n2} c_2 & \ldots & c_n r_{nn} c_n \end{bmatrix}$$

The elements of $[A_p]$ are

$$\begin{aligned} a_{pij} &= c_i r_{ij} c_j + d_i r_{ij} d_j \\ &= \frac{\cos\delta_i \cos\delta_j}{V_i V_j} r_{ij} + \frac{\sin\delta_i \sin\delta_j}{V_i V_j} r_{ij} \\ &= \frac{r_{ij}}{V_i V_j}\cos(\delta_i - \delta_j) \end{aligned}$$

Similarly,

$$b_{pij} = \frac{r_{ij}}{V_i V_j}\sin(\delta_i - \delta_j)$$

Equation (2.17) becomes

$$P_l = [P]^T[A_p][P] - [P]^T[B_p][Q] + [Q]^T[B_p][P] + [Q]^T[A_p][Q] \tag{2.18}$$

Again at any bus the following can be written

$$P_i = P_{Gi} - P_{Di}$$

where,

P_{Gi} = power generation at bus i

P_{Di} = power demand at bus i

Similar relation also holds for the reactive part.

Using this relation, Equation (2.18) can be rewritten as

$$\begin{aligned} P_l = &\left([P_G]^T - [P_D]^T\right)A_p\left([P_G]-[P_D]\right) - \left([P_G]^T - [P_D]^T\right)B_p\left([Q_G]-[Q_D]\right) \\ &+ \left([Q_G]^T - [Q_D]^T\right)B_p\left([P_G]-[P_D]\right) + \left([Q_G]^T - [Q_D]^T\right)A_p\left([Q_G]-[Q_D]\right) \end{aligned} \tag{2.19}$$

After expansion Equation (2.19) becomes

$$\begin{aligned} P_l = &[P_G]^T[A_p][P_G] - [P_D]^T[A_p][P_G] - [P_G]^T[A_p][P_D] + [P_D]^T[A_p][P_D] - [P_G]^T[B_p][Q_G] \\ &+ [P_D]^T[B_p][Q_G] + [P_G]^T[B_p][Q_D] - [P_D]^T[B_p][Q_D] + [Q_G]^T[B_p][P_G] - [Q_D]^T[B_p][P_G] \\ &- [Q_G]^T[B_p][P_D] + [Q_D]^T[B_p][P_D] + [Q_G]^T[A_p][Q_G] - [Q_D]^T[A_p][Q_G] - [Q_G]^T[A_p][Q_D] \\ &+ [Q_D]^T[A_p][Q_D] \end{aligned} \tag{2.20}$$

This is the standard general transmission loss equation for real power. A similar expression for reactive power can also be obtained. This expression will be used in Chapter 4 for the formulation of the Marginal Transmission Loss Approach.

2.4 Role of Transmission Loss in Present and Future Models of Network.

In a monopoly system, transmission loss plays a key role in determining economic system operation. A utility runs its network for minimum operating cost. A load dispatch centre (LDC) decides the generation of committed units in a network such that the production cost is minimized. The cost associated with transmission loss is distributed and charged to the consumers.

Deregulation of electrical power market has modified the way transmission loss is treated. The allocation of transmission loss has become a contentious issue. Due to open access policy in some countries, utilities share a common transmission facility. Transmission losses occur due to the flow of power for all transactions and it is difficult to find the component of the loss occurring due to a particular transaction. Some work has been done to find a generator's contribution to a particular load or to a line flow [8,11]. Those works use proportional methods for separating the flows in a line and trace back to generators from loads to find a generator's contribution to load. Although these methods identify links between sources and sinks, these do not help much in the case of a bilateral contract between a seller and a buyer.

Several countries, that include England, Brazil, Canada and USA, are trying to implement the concept of deregulation. The existing systems are actually different combinations of old monopoly and predicted deregulated systems. In a deregulated world, however, utilities would be facing new competitors that could charge lower rates. Under deregulation, local utilities would have to allow competitive power producers to use their transmission lines. A local utility, typically the owner of a transmission facility, would charge other utilities who would use their transmission network.

In some regions, power systems are still controlled by single entities (SaskPower) [5]. They can buy power from independent generators. The controlling utility can easily determine how much power they would buy and the associated transmission loss. The transmission loss, resulting from the purchase of external power, can be summed up with the transmission loss occurring due to internal supply.

On their way to deregulation, some systems have introduced the idea of energy pool or independent system operator (ISO). The energy pool structure is a spot market through which demands for electricity are met on the basis of hourly price and volume bids by generators and buyers. The ISO or the pool matches the sellers and the buyers based on their bidding prices and supervises the overall system security and reliability. The energy pool keeps record of the transactions and allocates the transmission loss among the participating utilities. The challenge faced the power pool and the ISO is how to allocate the transmission loss and what should be the criterion for charging other utilities. As no unanimous approach exists, different systems are utilizing different methods to allocate transmission losses. Some of the methods are:

- Embedded cost pricing
- Cost Causation-based pricing
- Usage-based pricing
- Location-specific pricing
- Real-time pricing

- Congestion pricing
- Opportunity cost pricing
- Market-based pricing

In a deregulated environment, consumers have the choice to buy power from any generating companies. A consumer could take a decision by comparing the rates and services provided by different utilities. For the last 100 years, utilities have operated as sole monopolies, with varying degrees of price fixing and regulation from governments. In a deregulated environment, consumers could choose an electric company the way they now choose a long-distance telephone service.

The challenge arises immediately for the allocation of transmission loss among the competing utilities. As an individual buyer can have a bilateral agreement with any utility, the loss related to this supply of power should be taken into consideration in preparing the agreement. The transmission loss originating from a bilateral contract should be assessed in such a way that the assessment should be viewed as fair and transparent.

Power systems typically operate under slowly changing dynamic conditions that can be analyzed using quasi steady state analysis. Moreover, transmission systems operate under balanced or near-balanced condition allowing per phase analysis to be used with a high degree of confidence in the solution. These lead to the use of the conventional load flow technique as a more accurate and valid way of finding transmission network loss.

The proper calculation of transmission loss is very important for deregulated systems. The main objective of this calculation is to distribute the loss among different utilities in a fair and equitable manner. Transmission loss is associated with the supply of power to any load in a system. It is important to know the associated transmission loss required to provide power to each individual load. This fact requires that, in a deregulated environment, transmission loss should be calculated in terms of loads, instead of generations.

2.5 Deregulated Market Operation

The most common form of market at present is Power Pool due to its simple nature. The generating utilities or IPP and customers both bid for buying and selling power at the power pool. Power pool conducts different types of auction: day ahead market, hour ahead market, real time market etc. Markets in different deregulated power systems is operated by different organizations, such as Power Pool of Alberta is responsible for market operation in Alberta, Independent Market Operator (IMO) runs the power market in Ontario. Similar duties are performed by the ISO in New England and by the Nordic Power Exchange in Norway. These markets operate on either day-ahead or hour-ahead market policy.

In a pool system both generating utilities and customers bid for selling and buying electrical power. The generating utilities do not have any target for any specific customer rather they bid for getting access to the grid. A generating utility would be out of the competitive market if its price is too high and on the other hand a customer would have no power if its offer were too low. Thus the pool fixes a single price for every hour which is determined by basic supply demand relationship of economics.

2.5.1 Market Clearing Price

The market operators are responsible to maintain a balance between the supply and demand of power. An electric power system must have sufficient supply of power in order to meet the customer's requirement. The market operators determine the price of electricity from the data obtained for supply and demand of power. This is price of electricity is commonly known as market clearing price. The market clearing price is set from the supply and demand relation in such a way that all power demands would be satisfied. The companies who bid higher than the market clearing price will not be able to sell any power. All companies who bid less than the market clearing price are considered as successful bidders and will be considered for supplying the demand. All successful bidders will get paid the market clearing price irrespective of their bidding prices.

The competition among the suppliers of electric power is the soul of market economy. If a supplier bids very high price for power and market clearing price is below than its bid price, then this particular high-bidding supplier will not be included in the load dispatch schedule. This fact will force the supplier out of business. The fear of getting out of business encourages the supplier to bid the most competitive prices in order to compete for dispatch in the wholesale marketplace.

In Alberta, deregulation is introduced in January 1, 2001 [61]. According to their policy, electricity is purchased on a centralized basis by the Power Pool of Alberta. Generators bid on an hourly basis to supply the load demand. The Power Pool of Alberta determines the market clearing price from the supply curve of power and from the demand of total power. The demand side bidding has not been introduced in Alberta. Alberta's power demand varies from 4,500 to 8,000 MW. The following figure shows how the Power Pool of Alberta sets the market clearing price. Figure 2.2 depicts how energy price is set by Alberta Power pool for any particular hour. Demand in Figure 2.2 is the total demand of the system and a vertical line represents the demand. Generators' bids are stacked. The intersection of demand line and the stepped bid curve determines the market clearing price. Similar graphs are used to set market clearing price for every hour of a day.

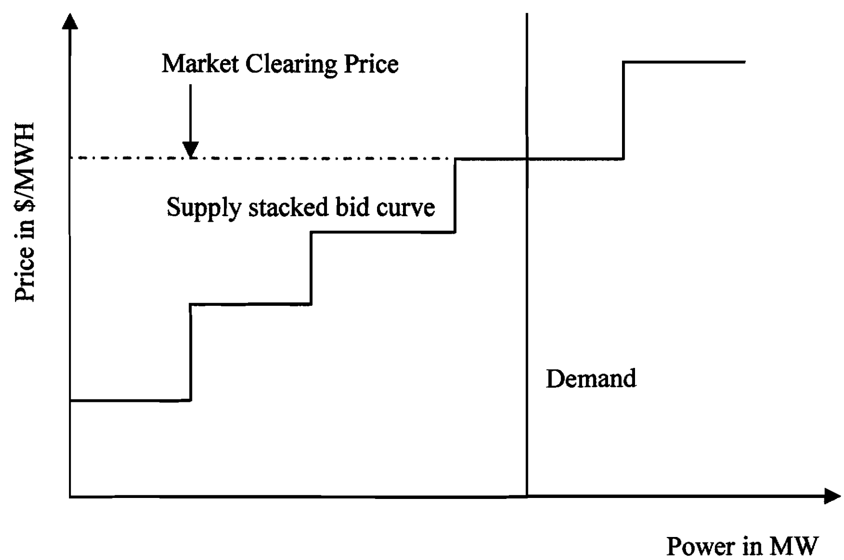

Fig. 2.2: Determination of market clearing price in Alberta Power Pool.

In Ontario, the IMO sets the limits on upper and lower boundaries of energy price [62]. The generating utilities submit their offer for every hour of a single day. The offer includes both quantity and price of power. The IMO starting from the lowest offer stacks up offers in an ascending order until total power supply meets the total demand. The market clearing price is set in similar way as done in Alberta which is based on the last accepted offer where it meets the actual load demand. All successful suppliers get the same price which is the market clearing price.

In New England, market operation is similar to that of Ontario. New England ISO uses day-ahead-hourly market strategy [63]. Day-ahead-hourly market means that generating utilities and electricity suppliers separately bid the day before for every hour of the day. The ISO gathers the bids and stack them from lowest to highest and match the forecasted load demand for every hour of the day in consideration. The highest bid offer from the stepped bid curve that meets the power demand sets the market clearing price for electricity for a particular hour of the day. This is the price that will be paid to all suppliers.

In Norway, market operation is similar to those of North American markets but in the Nordic market sellers as well as buyers submit their bids for selling and buying power respectively. Participants offer their bids for next-day physical delivery of power at Nord Pool's spot market [64] and hence the market is referred to as day-ahead market. The market operates on the principle of bidding for selling and buying power of one hour for 24 hours of the next day. The dead line is noon of everyday for submission of bidding for the next day. After the dead line is over Nordic Power Exchange collects all bids and prepares two curves: an aggregate demand curve and an aggregate supply curve. The market clearing price is determined by the intersection of these two aggregate supply and demand curves. Figure 2.3 shows supply-demand curve and market clearing price.

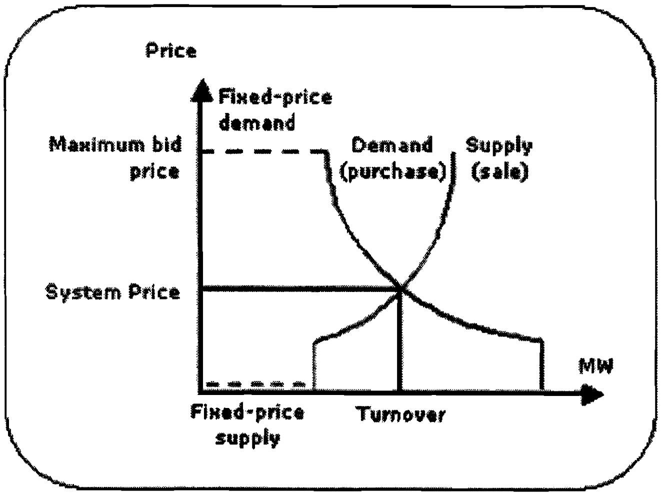

Fig. 2.3: Market clearing price in Nordic Power Pool (Courtesy http://www.nordpool.no/information/reports/nordic_market_report/chapter_004.html).

2.6 Bilateral Contract Options in Deregulated Power Systems

In a deregulated power system generating utilities and customers may sign contracts for selling and buying of power in addition to the existence of a power pool. These contracts would not affect any other contracts which are already in place. The concept of bilateral contracts allows the customers and generating plants to work according to their policy and does not make them dependent on the fluctuation of energy market. Bilateral contracts enable customers to make their best deals for energy supply with whoever in the competitive market is the most effective to meet their load demands. Power producers may choose to compete not only by price but also by contract duration, payment terms, type of generation and type of electric service. The price fixation and other services and particulars of the contract would be determined by the two parties involved in the contract. This would give them more liberty and flexibility of choices.

Any customer can choose any supplier for buying power in a deregulated system. According to a bilateral contract, a contracted generating unit would be responsible for supplying power to its contracted customer. The seller arranges the transportation of the contratced power over a transmission network. Since every eletrical transaction causes transmission loss, the generating unit should produce enough energy to cover the load demand of the customer as well as the associated transmission loss. A generating unit may however, consider different options for supplying power to its contracted load:

- The power producer may consider to contract the ISO for supplying its contracted load and associated loss. The power supplying utilities may consider this option when the market is low. Then it might be cheaper to buy power from market place and sell it to the contracted customer. A bilateral contract and the price set in a contract may be influenced by the reliability of power supply. In the case of a firm supply, the ISO will make sure that the contracted load will be out of any kind of load curtailment schedule. The ISO may curtail load of other customers but a bilaterally contracted customer will remain connected.

- The power producer may consider to supply the contracted load and ask the ISO for providing the associated loss. The power producer will pay the ISO for the energy to make up for the transmission loss according to the market price. The generator may prefer to supply power to the contracted load by itself when it does not want to fully rely on the market for buying everyday-power. In addition the generating plant will have the responsibilty for providing reliable power.

- The power producer may prefer to supply the load and the associated transmission loss by itself. The power producer in this case would produce enough power to cover for its contracted load as well as the resulting transmission loss and utilize the transmission network under the jurisdiction of an ISO to transport the energy to the contracted load point. The power producer have to pay the ISO a network access fee for the usage of the transmission network.

CHAPTER 3: INCREMENTAL LOAD FLOW APPRAOCH

3.1 Incremental Load Flow Approach

Transmission loss is a function of system configuration and it varies with load and generation. For a fixed system configuration loss varies with load and generation. The system configuration of a power system network usually remains unchanged unless lines are taken out of service due to a fault or maintenance or added. As a result, transmission loss could be easily determined with the help of a load flow analysis.

A conventional load flow analysis is performed to obtain quantities like line flows, line losses, bus voltages and bus angles. It does not however, provide the share of transmission loss of a particular generator in a system. Some methods have been reported to find the individual contribution of a load or a generation in the aggregate [33,34,65]. These methods, too, are based on conventional load flow analyses. These methods, however, do not validate the concept of deregulation. Those analyses do not take bilateral contracts between buyers and producers of electrical power into account.

In a deregulated environment, a generator who enters into a bilateral contract would be responsible for supplying energy to its contracted consumer as well as the loss associated with the demand. In a typical system there would be a number of bilateral agreements. A modified load flow analysis, named as the Incremental Load Flow Approach (ILFA), has been developed to determine the loss associated with an individual transaction. The technique is explained in the following.

In the ILFA, a load flow program is run for load levels from zero to their given level for each load under a bilateral contract in a system in a sequential manner. Loads are increased by a small increment in every iteration. Each generator is assumed to have a fixed consumer or load in the system and supposed to produce the power to meet the load demand of its customer and the associated loss. When a certain load is increased by a pre-specified increment, the corresponding increase in transmission loss is assigned to the generator that is in contract with this particular consumer. In other words, the generator (Generator X), which has a bilateral agreement with a consumer (Consumer A), would be responsible for the load demand (of Consumer A) and the loss originating from the contract.

The increased transmission loss for an incremental change in a given load is calculated and is assigned to a corresponding generator who is responsible for supplying the load. During an iteration, only one load is incremented while other loads are held constant.

3.2 Assumptions in the ILFA

A load flow technique, in general, requires that the power generation for PV buses be specified. According to the ILFA, the generation of a PV bus should be the sum of its load and its share of the total line loss. This share would be calculated from the incremental transmission loss. The loss is unknown prior to the first iteration. To overcome this difficulty in the first iteration, the associated loss is neglected and the generation of a PV bus is made equal to load. This would not affect the result as long as the increment size is kept small.

After the first iteration, the generation at a PV bus can be assigned as the sum of its discrete incremental load and the respective transmission loss from the previous iteration. This means that the assigned loss is always lagging behind the actual loss by one step. The difference would be very small if the step size is kept relatively small.

It has been assumed that each load has a constant reactive to real power ratio. This reactive ratio, the ratio of reactive to real power, is defined as,

$$\mu = \frac{\text{reactive power}}{\text{real power}}$$

This assumption reduces the complexity of calculations involved in the ILFA. The reactive ratio would depend on the nature of the load and may vary from customer to customer.

3.3 Example System

A small hypothetical system has been considered in this section for the purpose of numerical examples related to the allocation of transmission loss.

The hypothetical system consists of six buses with two generators and two loads. The generators and loads represent bulk producers and bulk consumers. The loads and the generators are connected through five lines. Two tap-changing transformers are connected on the load sides. The system is shown in Figure 3.1.

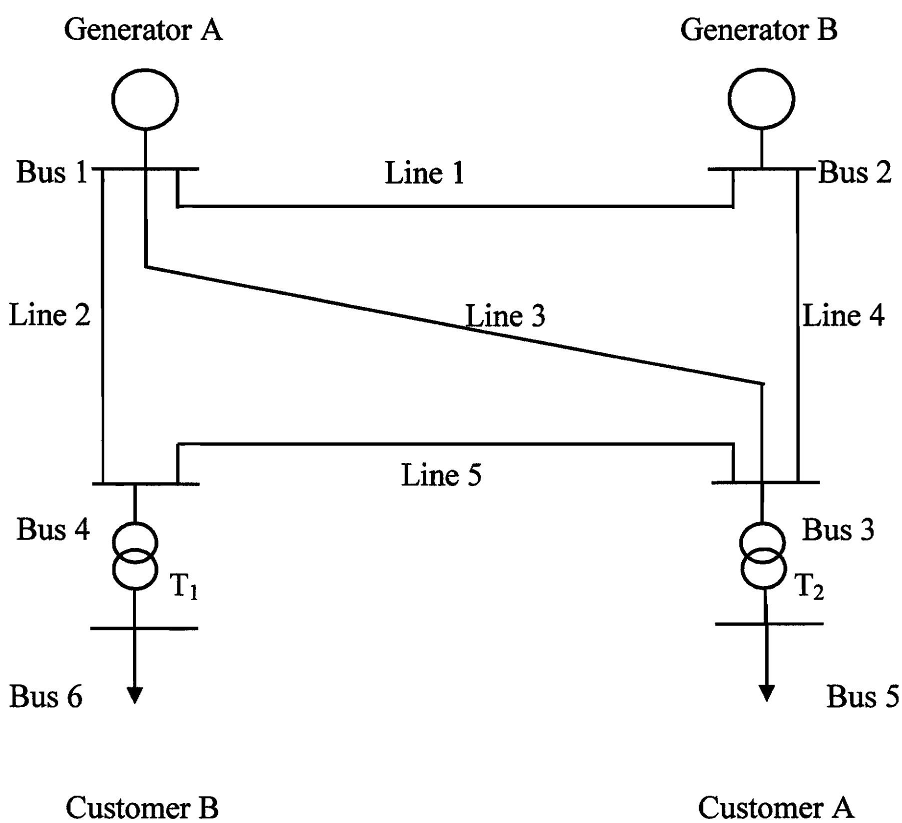

Fig. 3.1: Test system network.

Generators A and B are connected to Buses 1 and 2 respectively. Customers A and B are connected to Buses 5 and 6 respectively. Two tap changing transformers T_1 and T_2 are connected between Buses 4 - 6 and 3 - 5 respectively. Under the existing situation, the total load could be supplied by the combined generation of Generators A and B. But the goal is to investigate the effect of the condition where each generator is tied to its own load by virtue of a contract. In a deregulated system, the concept of region does not exist and hence the customers are free to choose their own utilities. The objective is to find the share of transmission loss due to bilateral contracts between generating utilities and customers. For this purpose it has been assumed that bilateral contracts exist between Generator A and Customer A and between Generator B and Customer B. As a result of the contracts, Generator A will supply the load demand of Customer A only, and Generator B will meet the demand of Customer B. As the generators are obliged to supply certain amount of loads to their respective customers they are also supposed to satisfy the associated transmission losses. The unknown at this point is the loss that an individual generator is responsible for.

The system parameters of the hypothetical system are shown in Tables 3.1 - 3.3. Bus 1 has been considered as the swing bus except in Case 1. The base values are 100 MVA and 138 kV. The tap changing transformers are set at nominal setting of 1 initially.

Table 3.1: Line parameters.

Line Number	From Bus	To Bus	Length km	Resistance (p.u.)	Reactance (p.u.)
1	1	2	80	0.0157	0.0787
2	1	4	50	0.0105	0.0525
3	1	3	120	0.0367	0.1837
4	2	3	50	0.0105	0.0525
5	3	4	100	0.0262	0.1312

Table 3.2: Transformer data.

Transformer Number	From Bus	To Bus	Resistance (p.u.)	Reactance (p.u.)
1	4	6	0.0053	0.0367
2	3	5	0.0039	0.0315

Table 3.3: Generating unit characteristics.

Generating Unit	Cost Function ($/hr)	Maximum Output (MW)	Minimum Output (MW)
A	$0.022P_1^2+12.45P_1+70$	500	90
B	$0.024P_2^2+13.65P_2+80$	400	40

3.4 Allocation of Transmission Loss by Using the ILFA

In order to appreciate loss allocation in a deregulated system, different base cases have been studied first with the help of a conventional load flow analysis. Base cases assume that the hypothetical system contains only one load and one generator. Base case load flow analyses help to find individual transmission loss for each generator. After the base cases, the system has been analyzed for two loads and their corresponding generators with the help of conventional load flow technique. This helps to find the effect of combined load on transmission loss and generations. Finally, the Incremental Load Flow Approach has been utilized to determine the share of transmission loss of the generators.

3.4.1 Case 1

At this stage, it is assumed that the hypothetical system has a generator (Generator B) and a load (Customer B). Generator B has been in contract with Customer B for supplying its demand. Two different load situations have been considered and the conventional load flow program has been used for these two load levels for finding the transmission loss. The corresponding generations and line losses are shown in Tables 3.4 and 3.5.

Table 3.4: Real and reactive power generation and line loss for Case 1.

	L_a (p.u.)	L_b (p.u.)	G_a (p.u.)	G_b (p.u.)	Line loss (p.u.)
Real	0	1.5	0	1.5799	0.0799
Reactive	0	0.9	0	1.3412	0.4412

Table 3.5: Real and reactive power generation and line loss for Case 1 with reduced load.

	L_a (p.u.)	L_b (p.u.)	G_a (p.u.)	G_b (p.u.)	Line loss (p.u.)
Real	0	0.9	0	0.9243	0.0243
Reactive	0	0.54	0	0.6745	0.1345

It has been mentioned earlier that Generator B is operating only to meet the demand of Customer B. Table 3.4 shows that for a load of 1.5 + j0.9 p.u. the required generation is 1.5799 + j1.3412 p.u. The associated line loss is 0.0799 + j0.4412 p.u. The load of Customer A is kept at zero and hence the generation of Generator A is also forced to zero. This is due to the fact that Generator A is contractually obligated to supply Customer A.

The load of Customer B has been changed to 0.9 + j0.54 p.u. The corresponding generation is 0.9243 + j0.6745 p.u. and the line loss is 0.0243 + j0.1345 p.u. (Table 3.5).

3.4.2 Case 2

This case has been investigated to see the effect of the bilateral contract between Generator A and Customer A on the associated transmission loss. The load of Customer B is set to zero and this allows the Generator B to produce nothing.

The generation and the line loss due to this contract is shown in Table 3.6.

Table 3.6: Real and reactive power generation and line loss in the system for Case 2.

	Load L_a (p.u.)	Load L_b (p.u.)	Generation G_a (p.u.)	Generation G_b (p.u.)	Line loss (p.u.)
Real	1.5	0	1.5527	0	0.0527
Reactive	0.9	0	1.2057	0	0.3057

For a load of 1.5 + j0.9 p.u. by Customer A, the Generator A is producing 1.5527 + j1.2057 p.u. and the calculated line loss is 0.0527 + j0.3057 p.u.

3.4.3 Case 3

After having the load flow solutions for individual contracts between the generators and the customers, both generators and loads have been brought into the system, which is our original system under consideration. Two different load situations have been considered - Customer A has same load in both situations while Customer B has different loads. Due to the bilateral contracts, Generator A would provide power to Customer A and Generator B would provide power to Customer B.

The generations of individual units and total line losses obtained from the conventional load flow program have been presented in Tables 3.7 - 3.10. The conventional load flow study needs the generation of PV buses to be specified and these generations are obtained from Cases 1 and 2. Table 3.7 shows the real and reactive power generations and line loss. Bus 1 is considered as the slack bus and the loads are assumed to be equal. Data obtained from Case 1 is used as the generation data G_b.

Table 3.7: Power generations and line loss in the system for Case 3.a with Bus 1 as the slack bus.

	Load L_a (p.u.)	Load L_b (p.u.)	Generation G_a (p.u.)	Generation G_b (p.u.)	Line loss (p.u.)
Real	1.5	1.5	1.5187	1.5799	0.0986
Reactive	0.9	0.9	1.4679	0.9036	0.5716

Table 3.8 shows the real and reactive power generations and line loss where Bus 2 is considered as the slack bus. The loads are assumed to be equal and generation data G_a is taken from Case 2.

Table 3.8: Power generations and line loss in the system for Case 3.a with Bus 2 as the slack bus.

	Load L_a (p.u.)	Load L_b (p.u.)	Generation G_a (p.u.)	Generation G_b (p.u.)	Line loss (p.u.)
Real	1.5	1.5	1.5527	1.5457	0.0984
Reactive	0.9	0.9	1.4615	0.9090	0.5705

Table 3.9 shows the real and reactive power generations and line loss. Bus 1 is considered as the slack bus and the loads are assumed to be unequal. Data obtained from Case 1 is used as the generation data G_b.

Table 3.9: Power generations and line loss in the system for Case 3.a with Bus 1 as the slack bus.

	Load L_a (p.u.)	Load L_b (p.u.)	Generation G_a (p.u.)	Generation G_b (p.u.)	Line loss (p.u.)
Real	1.5	0.9	1.5354	0.9244	0.0597
Reactive	0.9	0.54	0.9068	0.8841	0.3509

Table 3.10 shows the real and reactive power generations and line loss where Bus 2 is considered as the slack bus. The loads are assumed to be equal and generation data G_a is taken from Case 2.

Table 3.10: Power generations and line loss in the system for Case 3.a with Bus 2 as the slack bus.

	Load L_a (p.u.)	Load L_b (p.u.)	Generation G_a (p.u.)	Generation G_b (p.u.)	Line loss (p.u.)
Real	1.5	0.9	1.5527	0.9071	0.0598
Reactive	0.9	0.54	0.9039	0.8875	0.3514

The transmission losses shown in Table 3.7 have been calculated by considering Bus 1 as the slack bus. Both loads are equal to 1.5 + j0.9 p.u. and as evaluated in Case 1, the magnitude of G_b has been considered as 1.5799 p.u. Bus 2 has been considered as the slack bus and G_a has been taken as 1.5527 p.u. as evaluated in Case 1 (for the transmission losses shown in Table 3.8.). The real and reactive line losses are less than the sum of the losses found in Cases 1 and 2. But, the generation of reactive power does not drop for each generator. As for example, Table 3.8 shows that the reactive power generation of A (1.4679 p.u.) has been increased while the opposite has happened to B. Reactive generation of B has dropped to 0.9036 p.u. which is slightly higher than the reactive load of Customer B. This shows clearly that Generator B is not producing as much reactive power as it produced under Case 1. This is due to the fact that transmission losses, both real and reactive are nonlinear functions of load and generation.

Tables 3.9 and 3.10 shows the generation and loss for unequal demands of Customers A and B. Customer B's load (L_b) has been changed to 0.9 + j0.54 p.u. while Customer A's load (L_a) remains unchanged at 1.5 + j0.9 p.u. The evaluations have been done with Bus 1 as the slack bus (Table 3.9) and then Bus 2 as the slack bus (Table 3.10) and corresponding PV bus generations have been obtained from Cases 1 and 2. Comparing the results shown in Table 3.6 with Tables 3.9 and 3.10, Generator A is producing much less reactive power than it produces under Case 2. It is clear from Table 3.9 that an increased burden of producing reactive power has been taken up by Generator B (0.8841 p.u.) while Generator A (0.9068 p.u.) is producing less than that produced in Case 2 which is just higher than the reactive demand of Customer A. This is due to the fact that the calculated values of required generations would vary if the slack bus is changed. In Tables 3.11 and 3.12, the generation, total transmission loss and the contribution of the generators to transmission loss have been summarized.

The situations considered in Case 3 lead to a set of questions. What should be the appropriate share of each generator in producing real and reactive power? What would happen if one of the generators is unable to produce its due share and then to whom would this generator be liable for this support? The incremental load flow approach can be utilized to find the appropriate share of each generator in terms of real and reactive power.

Table 3.11: Comparison of real loss contribution for the three different cases.

	Load (p.u.)		Slack Bus	System Loss (p.u.)	Generation to mitigate loss (p.u.)	
	L_a	L_b			G_a-L_a	G_b-L_b
Case 1.a	0	1.5+j0.9	2	0.0799	0	0.0799
Case 2	1.5+j0.9	0	1	0.0527	0.0527	0
Case 3.a	1.5+j0.9	1.5+j0.9	1	0.0986	0.0187	0.0799
			2	0.0984	0.0527	0.0457
Case 1.b	0	0.9+j0.54	2	0.0243	0	0.0243
Case 3.b	1.5+j0.9	0.9+j0.54	1	0.0597	0.0354	0.0244
			2	0.0598	0.0527	0.0071

Table 3.12: Comparison of reactive loss contribution for the three different cases.

	Load (p.u.)		Slack Bus	System Loss (p.u.)	Generation to mitigate loss (p.u.)	
	L_a	L_b			G_a-L_a	G_b-L_b
Case 1.a	0	1.5+j0.9	2	0.4412	0	0.4412
Case 2	1.5+j0.9	0	1	0.3057	0.3057	0
Case 3.a	1.5+j0.9	1.5+j0.9	1	0.5716	0.5679	0.0036
			2	0.5705	0.5615	0.0090
Case 1.b	0	0.9+j0.54	2	0.1345	0	0.1345
Case 3.b	1.5+j0.9	0.9+j0.54	1	0.3509	0.0068	0.3441
			2	0.3514	0.0039	0.3475

3.4.4 Case 4

In the ILFA, the loads under bilateral contracts are increased in incremental steps in a sequential manner. In any iteration, only one customer load is increased while the other loads are held fixed. Let us assume that load L_a is increased by a step of ΔL_a while L_b stays at its previous level. A load flow program has been developed to assess the generations and transmission losses for this condition. Since load demand of Customer B (L_b) is unchanged, the resulting incremental transmission loss becomes the obligation of Generator A. Generator A has to produce adequate power to support the incremental load demand of customer A and the resulting incremental transmission loss. In the next iteration, L_b has been increased by a step size of ΔL_b while L_a remains fixed. Again, generations and transmission losses are calculated and the incremental loss is assigned to Generator B.

In order to obtain a load flow solution, the buses connected with generating units have to be declared as voltage controlled (PV) bus. The power generations at these buses are required to be specified prior to the load flow. In a bilateral contract generation of the PV bus should be the sum of load and its share of the total line loss. The magnitude of this line loss, however, is unknown before the load flow analysis can be performed.

Generator B is connected to Bus 2 and as such has been declared as a PV bus. The generation at Bus 2 has been calculated on the basis of current load (L_b) and transmission loss calculated from the previous load. Since the transmission loss for the first increment of load L_b is unknown, the required output of Generation B (G_b) has been specified as ΔL_b. In the next increment of Customer B's load , G_b has been updated as L_b plus the corresponding share of the line loss from the previous iteration. This approximation works well as long as the step size remains small. A step size of 1 MW is considered in this case.

Using the above mentioned approximation and reactive ratio of 0.6, the incremental load flow program has been utilized to obtain generations and transmission losses. Individual generations and transmission losses are shown in Tables 3.13 and 3.14.

Table 3.13: Individual generations and total transmission loss obtained from ILFA for equal load condition.

	L_a (p.u.)	L_b (p.u.)	G_a (p.u.)	G_b (p.u.)	Line loss (p.u.)
Real	1.5	1.5	1.5414	1.557	0.0984
Reactive	0.9	0.9	1.4635	0.9072	0.5707

Table 3.14: Individual generations and total transmission loss obtained from ILFA for different load condition.

	L_a (p.u.)	L_b (p.u.)	G_a (p.u.)	G_b (p.u.)	Line loss (p.u.)
Real	1.5	0.9	1.5414	0.9182	0.0597
Reactive	0.9	0.54	0.9057	0.8853	0.3511

The main goal of the ILFA is to allocate transmission losses among the generating utilities with bilateral contracts in a deregulated power system network. Six different situations have been considered for this purpose and the corresponding transmission loss allocations have been assessed.

3.4.5 Unequal Load and Different Reactive Ratio

In this section, unequal reactive ratios as well as the unequal real loads for Customers A and B are considered. The reactive ratio for Customer B has been varied while maintaining a constant reactive ratio for Customer A. Three different constant reactive ratios for Customer A have been considered in this evaluation. Transmission loss allocations are shown in Table 3.15. The real load demands of Customer A and B are 1.5 and 0.9 p.u. respectively.

Table 3.15: Calculated share of transmission loss of Generators A and B for unequal load and unequal reactive ratio.

Real L_a (p.u.)	Real L_b (p.u.)	Reactive ratio μ_a	Reactive ratio μ_b	Calculated Loss Share of A (p.u.)		Calculated Loss Share of B (p.u.)	
				Real	Reactive	real	Reactive
1.5	0.9	0.6	0.3	0.0397	0.239	0.0148	0.0836
1.5	0.9	0.6	0.4	0.0401	0.2407	0.0157	0.0899
1.5	0.9	0.6	0.5	0.0405	0.2427	0.0171	0.0974
1.5	0.9	0.5	0.3	0.0359	0.2156	0.0147	0.0832
1.5	0.9	0.5	0.4	0.0363	0.217	0.0156	0.0894
1.5	0.9	0.4	0.3	0.0328	0.1965	0.0147	0.0825

Figures 3.2 - 3.5 show loss allocations for different combinations of real loads and reactive ratios.

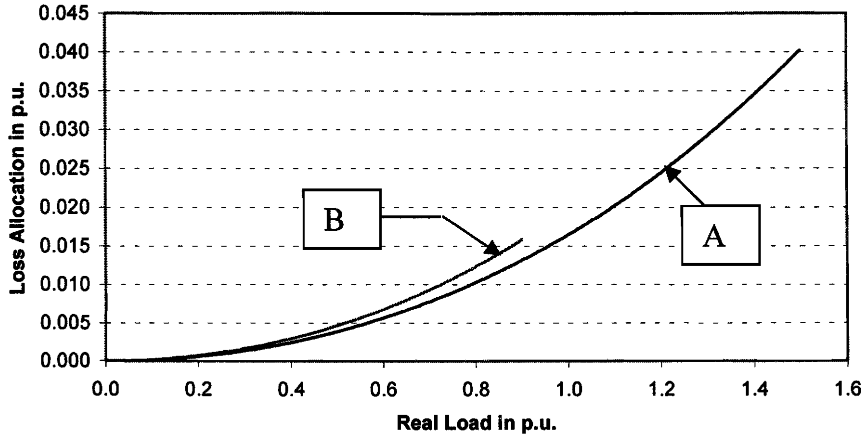

Fig. 3.2: Loss allocation of real power for Generators A and B for unequal demands (μ_a=0.6, μ_b=0.4).

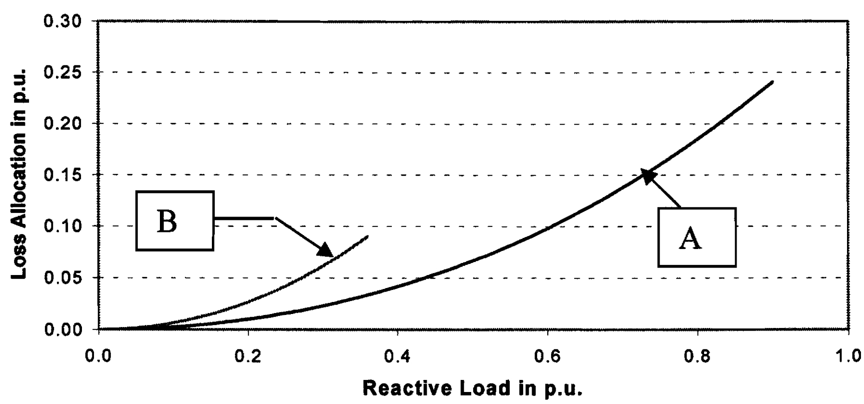

Fig. 3.3: Loss allocation of reactive power for Generators A and B for unequal demands (μ_a=0.6, μ_b=0.4).

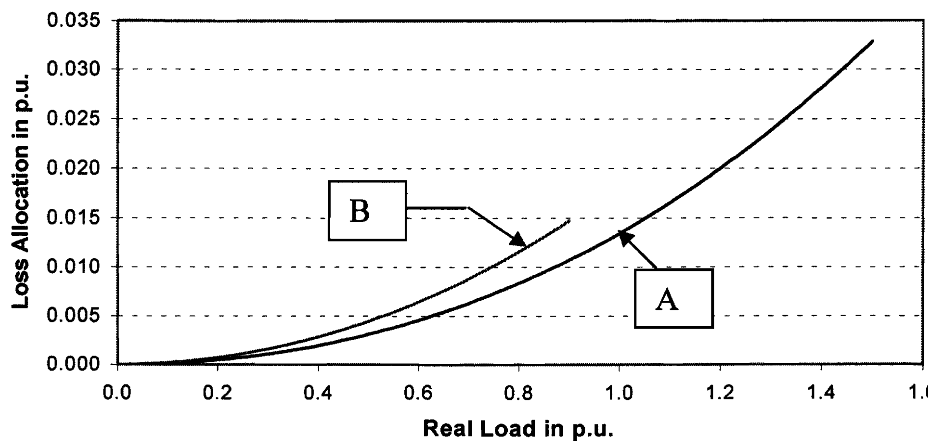

Fig. 3.4: Loss allocation of real power for Generators A and B for unequal demands (μ_a=0.4, μ_b=0.3).

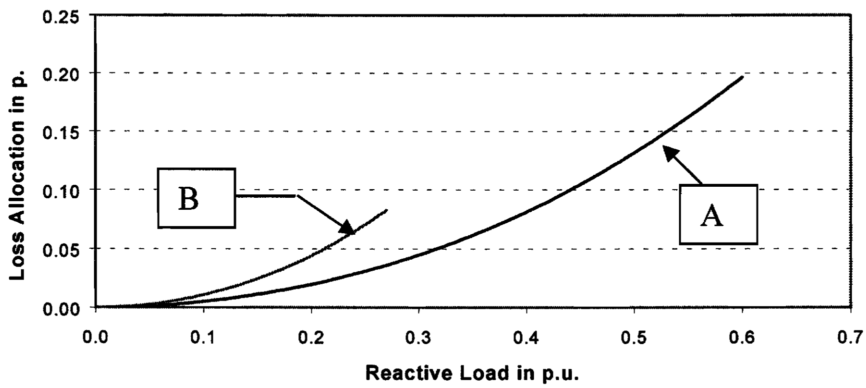

Fig. 3.5: Loss allocation of reactive power for Generators A and B for unequal demands (μ_a=0.4, μ_b=0.3).

The nature of the loss allocation curves remains similar for variations in load demands and reactive ratios. In both cases of reactive ratios, real loss allocation of Generator A follows that of B closely whereas reactive loss allocations differ significantly.

3.5 IEEE-Reliability Test System

The IEEE 24 bus Reliability Test System (RTS) [66] has been utilized to find to apply the developed methods for transmission loss allocation. The system is described below:

Generating and Reliability Data: The system contains 32 generating units ranging from 12 MW to 400 MW. The generating system includes thermal, fossil-oil, fossil-coal, nuclear and hydro units. Reliability data (mean time to failure, mean time to repair, forced outage rate) of the units are provided in Appendix- A. Appendix-A also contains the operating costs of the units.

Transmission System: The transmission system consists of 24 buses. These buses are connected by 38 lines and transformers. The transmission system has two voltage levels – 138 kV and 230 kV, and includes cables and overhead lines. Appendix-A provides transmission system data which includes line length, impedance, susceptance and ratings.

The transmission network has voltage corrective devices at Bus 14 (synchronous condenser) and at Bus 6 (reactor). These devices increase the network performance (maintaining rated voltage) under contingency conditions.

IEEE 24 bus Reliability Test System is shown in Appendix-A.

3.5.1 Bilateral Contracts in the IEEE-RTS

Transmission loss allocation can be found based on individual bilateral contracts in a deregulated power system network [48]. In practice, a power system network might operate in a mixed-mode system - a combination of power pool and bilateral contracts.

The previous six-bus test system was assumed to be completely deregulated which means only bilateral contracts exist in the system. These contracts are assumed to be among the generators and bulk customers. Transmission loss for individual contracts has been calculated using ILFA.

The IEEE-RTS is a moderately large system with a generating capacity of 3405 MW and 17 load points in the network. The customers connected to these 17 buses are bulk customers. It has been considered that a power pool is working for managing the energy requirements of the IEEE-RTS network. This power pool arranges the auction market and maintains the system security. In addition to the power pool, bilateral contracts are assumed to exist in this network.

The group of Generating units at Bus 7 is in a bilateral contract with the customer at Bus 9. It is termed as contract A. At Bus 7 there are three 100 MW units and these plants are already selling some energy to the pool. They have a contract (Contract-A) with the customer at Bus 9 for supplying 176 MW of real power and 36.21 MVAR of reactive power. Another bilateral contract (Contract-B) exists between the generating units at Bus 23 and the customer at Bus 19. At bus 23, there are two 155 MW and one 350 MW generators with a total capacity of 610 MW. These generators are also supplying the pool and has a bilateral contract with the customer at Bus 19 for supplying 181+j37 MVA of apparent power.

3.5.2 Loss Allocation in the IEEE-RTS Using the ILFA

Incremental Load Flow Approach (ILFA) can be utilized to determine the transmission loss share in a mixed-mode power system network. Before implementing the ILFA it is required to find the base condition or condition prior to the bilaterally contracted loads are added to the system. Base condition actually represents the power pool operations – power demand and supply in the network.

3.5.3 Power Pool Operation in the IEEE-RTS

Generating utilities or IPPs bid for selling their power at the power pool. The generating utilities do not have any target for any specific customer. Electricity is purchased on a centralized basis by a hypothetical Power Pool operator called Independent System Operator in many jurisdictions who manages the power balance of IEEE system. Generators bid on an hourly basis to supply the their energy. The Power Pool determines the market clearing price from the supply curve of power and from the demand of total power. The demand side bidding has not been considered in the IEEE system. The power demand in the system for a particular hour is considered to be 2494 MW of real power 589 MVAR of reactive power. Generators' bidding for this hour in the IEEE power pool is shown in Table 3.16.

Table 3.16: Generators' bidding in the IEEE power pool.

Bus	Size (MW)	No. of Units	Total Power (MW)	Price($/MW-Hr)
22	50	6	300	25
18	400	1	400	35
21	400	1	400	40
23	155	2	310	47
15	155	1	155	50
16	155	1	155	53
1	76	2	152	65
2	76	2	152	65
7	100	1	100	67
13	197	2	394	70

1	20	2	40	70
2	20	2	40	74
13	197	1	197	85
15	12	5	60	90

Figure 3.6 shows the bidding curve obtained from the generator side bidding in the IEEE system. Generators' bids are stacked and the curve has a stair like shape. Demand in Figure 3.6 is the total real power demand of the system and a vertical line represents the demand. The intersection of demand line and the stair-like bid curve determines the market clearing price which is 70 $/MW-Hr in this case. Similar graphs are used to set market clearing price for every hour of a day.

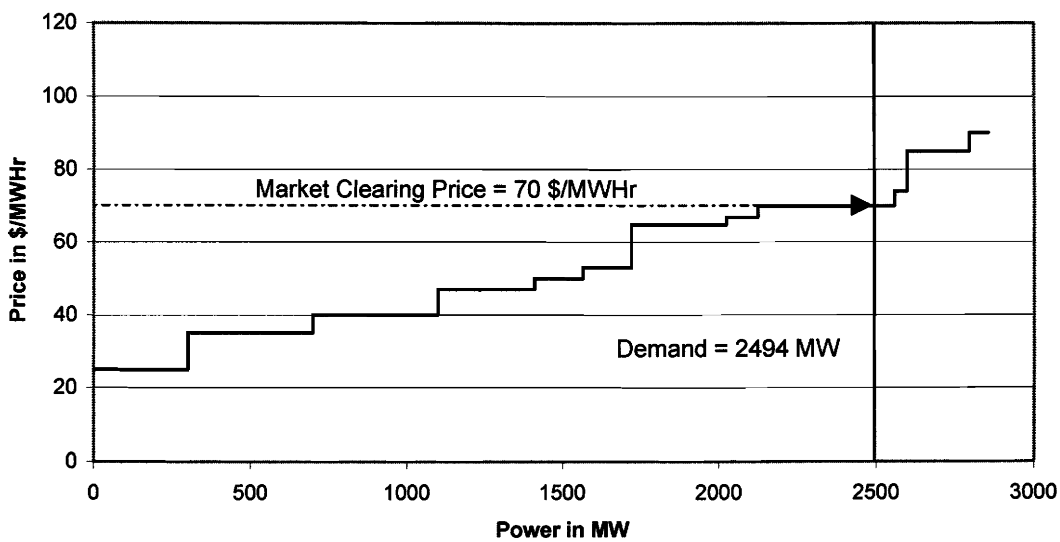

Fig. 3.6: Market clearing price in IEEE system.

Before the addition of the contracted load the demand in the network is 2494 MW of real power 589 MVAR of reactive power. This demand is supplied by the generators in the network including the units connected to Bus 7 and Bus 23. The load demands and power generations at various buses are listed in Table 3.17. Table 3.17 shows data in per unit;100 MVA and 138 kV have been chosen as base values for the system.

Table 3.17: Loads and Generations in the IEEE-RTS for the base case.

Bus	Load (p.u.)		Generation (p.u.)	
	Real	Reactive	Real	Reactive
1	1.0800	0.2200	1.5753	0.2075
2	0.9700	0.2000	1.7200	0.0511
3	1.8000	0.3700		
4	0.7400	0.1500		
5	0.7100	0.1400		
6	1.3600	1.1000		
7	1.2500	0.2500	1.0000	0.9520
8	1.7100	0.3500		
9				
10	1.9500	0.4000		
11				
12				
13	2.6500	0.5400	3.9400	1.0036
14	1.9400	0.3900		
15	3.1700	0.6400	1.5500	1.5847
16	1.0000	0.2000	1.5500	1.0991
17				
18	3.3300	0.6800	4.0000	0.6108
19				
20	1.2800	0.2600		
21			4.0000	-0.4205

22			3.0000	-0.3799
23			3.1000	-0.1185
24				

The total complex value of the generation in the system is 25.4353 + j4.5899 p.u. Transmission losses can be calculated from the generations and demands in the system. Total real and reactive power losses in the network are 0.4953 p.u. and –1.3001 p.u.

There are two bilateral contracts in the IEEE-RTS system. Contract-A is between load at Bus 9 and generators at Bus 7. Load at Bus 9 is termed as Customer A and generators at Bus 7 are termed Generator A. Second contract known as Contract-B is between load at Bus 19 and generators at Bus 23. Load at Bus 19 is termed as Customer B and generators at Bus 23 are termed Generator B.

In the ILFA, the loads are increased in incremental steps in a sequential manner. In any iteration, only one customer load is increased while the other loads are held fixed. Let us assume that load, L_9 is increased by a step of ΔL_9 while L_{19} stays at its previous level. A modified load flow program has been developed to asses the generation and transmission loss for this condition. Since load demand of Customer B (L_{19}) is unchanged, the resulting incremental transmission loss becomes the obligation of Generator A to produce it in order to support the incremental load demand of customer A. In the next iteration, L_{19} has been increased by a step size of ΔL_{19} while L_9 remains fixed. Again, generations and transmission losses are calculated and the resulting incremental loss is assigned to Generator B.

Generators A and B are connected to Buses 7 and 23 respectively have been declared as PV buses. An incremental step size of 1 MW has been used

The incremental load flow program has been utilized to obtain generations and transmission losses. Individual generations and transmission losses are shown in Tables 3.18 and 3.19.

Table 3.18: Contracted loads and generations in the IEEE-RTS determined by ILFA.

	L_9 (p.u.)	L_{19} (p.u.)	G_7 (p.u.)	G_{23} (p.u.)	Line loss (p.u.)
Real	1.76	1.81	2.6891	4.9138	0.4286
Reactive	0.3621	0.37	0.5624	0.0828	-1.5530

Table 3.19: Transmission loss allocation for the bilateral contracts in the IEEE-RTS.

Total Loss (p.u.)		Calculated Loss Share of A (p.u.)		Calculated Loss Share of B (p.u.)	
Real	Reactive	Real	Reactive	Real	Reactive
0.4286	-1.5530	-0.0706	-0.2836	0.0039	0.0306

The total real and reactive transmission loss become 0.4286 p.u. and –1.5530 p.u. respectively. According to Contract-A, generators connected to Bus 7 are delivering power to a load of 1.76+j0.3621 p.u. at Bus 9. Generators at Bus 7 are producing 1.000+j0.9520 p.u. for the pool in addition to what is required for the contracted load at Bus 9. Generators at Bus 7 are delivering a total of 2.6891+j0.5624 p.u. to the network. The related transmission loss shares of Contract-A are –0.0706 p.u. and –0.2836 p.u., real and reactive respectively.

Generators at Bus 23, involved in bilateral Contract-B, are delivering 3.1000-j0.1185 p.u. of power to the pool in addition to the contracted delivery at Bus 19. The generators are producing a total of 4.9138+j0.0728 p.u. The calculated transmission loss shares of Contract-B are 0.0090 p.u. and 0.0828 p.u., real and reactive respectively. It can be noted that the contracted generators are producing different reactive powers than they were producing before serving the contracted loads. It happened in order to maintain the system voltage profile within the prescribed limits by re-allocating the reactive power generations.

Transmission loss allocations for Contracts -A and -B are shown in Fig. 3.7 and 3.8.

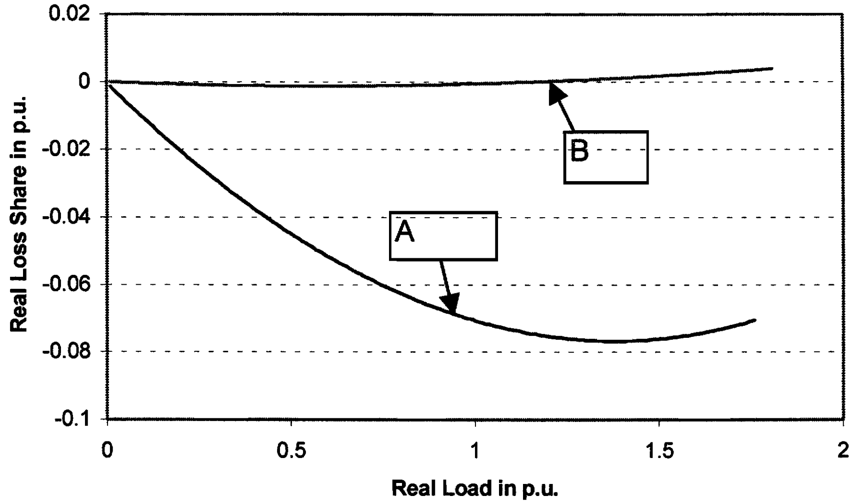

Figure 3.7: Real loss allocation for Contracts A and B plotted against real load.

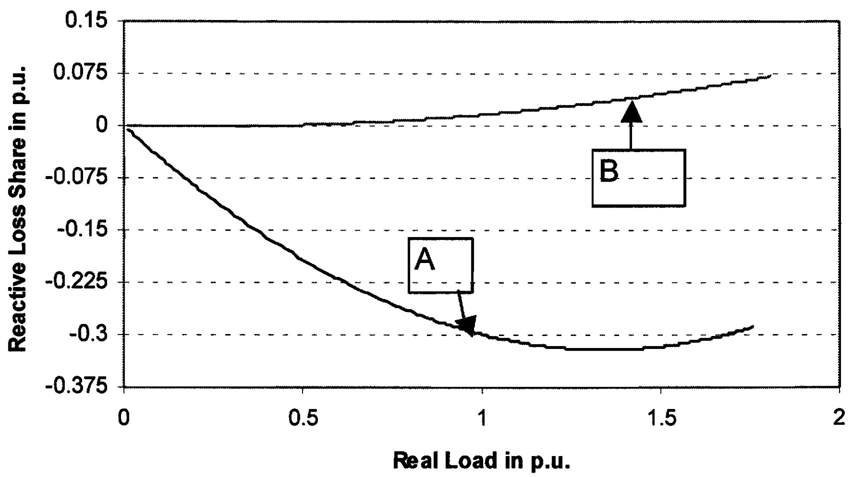

Figure 3.8: Reactive loss allocation for Contracts A and B plotted against real load.

Fig. 3.7 shows the real power transmission loss allocation for the contracts. It has been found that the final loss allocation for Contract-A is negative as shown in Fig. 3.7. The real loss allocation for Contract-A goes downward upto the point of 1.39 p.u. after which it begins to rise. Similar trend is found for the reactive loss allocation for Contract-A (Fig. 3.8). Real loss allocation for Contract-B stays positive as shown in

Fig. 3.7. Like its real counterpart, the reactive loss allocation for Contract-B also stays positive as shown in Fig. 3.8.

3.6 Overview of Loss Allocations

A detailed study has been done to find transmission loss allocation in a deregulated power system network. A modified load flow technique (ILFA) is employed to obtain losses, both real and reactive, for a bilateral contact in a network. A 6-bus test system has been utilized to find the transmission loss allocation where the system is considered to be deregulated. Only bilateral contracts are considered in this test system. Different load combinations have been taken into consideration and loss allocations have been calculated using the ILFA. Later IEEE 24-bus Reliability Test System is used to find the transmission loss allocation for bilateral contracts in a mixed-mode system. In the next chapter a generalized mathematical model is developed and utilized to obtain the transmission loss allocations.

A generator in a system may not be able to supply its due share of real and reactive power. Results obtained from the ILFA can be utilized to assess the shortfall of the generator. Real component of transmission loss is directly related to monetary issues and hence should be resolved in a fair way. The reactive loss distribution is also of importance, as it must be provided to maintain the system voltage level. The compensation for not producing enough reactive power, either for system constraints or own limitations, has been discussed in Chapter 6.

CHAPTER 4: GENERALIZED MARGINAL TRANSMISSION LOSS APPROACH

4.1 Transmission Loss in Power Systems

Power flow in a transmission network causes some transmission losses and it is quite significant in a large grid network. Transmission loss can be divided into two parts: real and reactive. Real power generated by a system should be equal to the sum of its loads and line losses. Reactive power in a system is required for system voltage stability. Reactive power loss must be provided for maintaining the safe voltage profile in a network. Unlike real power, reactive power does not have any direct monetary value but it is an extremely important factor in power system operation.

Total transmission loss can be calculated in different ways. One of the popular ways is to utilize Kron's [49] transmission loss formula. This formula is based on a number of assumptions and calculates transmission loss in terms of generations. Despite its many assumptions, Kron's formula gives a fairly close result when compared to the results obtained through more accurate methods [71]. But the advantage of this formula is that it is a function of generation only, and therefore, one can find approximate transmission loss in a system by knowing the generation of individual plants in a system.

4.2 General Transmission Loss Formula

In a monopoly utility system, generating units are committed and dispatched in a way that the total operating cost is minimized. Transmission loss is viewed as a part of the overall operating cost and included in the optimization process with the help of Lagrange multipliers. Under a bilateral contract agreements, in a deregulated environment, a generator is responsible for supplying power to its own customer. The Generator will use a common transmission facility managed by an Independent System Operator (ISO) to transport power to its customer. The ISO would provide equal access to all participating (competing) generating entities. Each generating entity that participates in a bilateral contract should produce enough power to meet its load and its share of transmission loss.

Transmission loss can be calculated accurately by AC load flow analysis. Due to the computational complexities of AC load flows, however other methods [72] have been developed to calculate transmission loss in a simpler way. A generalized transmission loss formula, mentioned in Chapter 2, that does not depend on network configuration can be used for calculating transmission loss in a convenient way.

Due to its application in economic optimization, transmission loss, in general, is expressed in terms of active power generations only. The George's formula [69] is the simplest one that is given by

$$P_l = \sum_{i=1}^{m} \sum_{j=1}^{m} (P_i B_{ij} P_j) \tag{4.1}$$

Where,

P_l = total transmission loss

P_i = active power injection at bus i.

m = number of generators

The coefficients B_{ij} are commonly referred to as loss coefficients. A more generalized formula is given by Kron. The Kron's Loss formula [49] is

$$P_l = K_{lo} + \sum_{i=1}^{m} B_{i0} P_i + \sum_{j=1}^{m} (P_i B_{ij} P_j) \tag{4.2}$$

In both formulae, active generations are used as the only variables to reduce the computational complexities. The main advantage of the generalized loss formulae is that they are easy to use and do not require iterations like load flows require.

It is clear from George's and Kron's formula that neither of them reflect the load situations in the network. Although the generations used in these formulae are based on load demand in the network, they do not provide sufficient information with respect to the transmission loss allocation of an individual generator within a bilateral contract in a deregulated network. In order to allocate transmission loss, the contribution of individual loads to power loss in a system has to be determined. Any change in a load should be taken into consideration and its effect should be reflected fully in transmission loss calculation. This way a generator under a bilateral contract should be able to ascertain the amount of power it is required to produce. It should generate enough power to satisfy its load plus meet its obligation for any additional transmission loss unless it has other special contracts with the ISO.

In order to allocate transmission loss among the generators in a fair way, a modified transmission loss formula is necessary which would allow one to find the responsibility of an individual generator who enters into a bilateral contract with a customer in a deregulated network.

4.3 Marginal Transmission Loss

The basic principle of the generalized loss formulae can be used in a modified form for the purpose of transmission loss allocation. The modified transmission loss formula should be a function of individual loads in a system and should be able to respond to changes in loads. As the generation of a supplier who enters into a bilateral contract should depend upon contracted load and its share of transmission loss, the modified loss formula would include generation in an indirect manner.

Assume a power system network where a supplier (Generator A) enters into a bilateral contract to supply energy to a specified customer (Load L_a). It is assumed that Generator A should produce enough power to supply Customer A and satisfy its share of transmission loss. Mathematically this can be written as

$$G_a = Load_a + Loss_a \tag{4.3}$$

Where,

G_a = active power generation of Generator A in MW,

$Load_a$ = active power demand of a bulk consumer in MW that Generator A is obliged to supply and

$Loss_a$ = Generator A's share of transmission loss in MW.

Equation (4.3) implies that generation is a function of load. A conventional loss formula uses the amount of generations as independent variables. Transmission losses are embedded in generations and are inseparable. But, in a deregulated system it is required to separate the loss that is originated due to a particular load. In a monopoly system, the losses are distributed as a consequence of economic operation. In a deregulated network, it is important and necessary to keep track of the losses arising from individual loads who enters into bilateral contract.

A modified loss equation has been derived which is a function of loads. The modified loss equation can be used to calculate loss associated with an individual load that enters into a bilateral contract with a supplier in a deregulated. It is assumed that the power produced by the contracted generator is transferred to its destination load using a deregulated network operated by an Independent System Operator (ISO).

4.4 Mathematical Model

Transmission loss in a network can be expressed as [68]:

$$\begin{aligned} P_l = {} & [P_G]^T[A_p][P_G]-[P_D]^T[A_p][P_G]-[P_G]^T[A_p][P_D]+[P_D]^T[A_p][P_D]-[P_G]^T[B_p][Q_G] \\ & +[P_D]^T[B_p][Q_G]+[P_G]^T[B_p][Q_D]-[P_D]^T[B_p][Q_D]+[Q_G]^T[B_p][P_G]-[Q_D]^T[B_p][P_G] \\ & -[Q_G]^T[B_p][P_D]+[Q_D]^T[B_p][P_D]+[Q_G]^T[A_p][Q_G]-[Q_D]^T[A_p][Q_G]-[Q_G]^T[A_p][Q_D] \\ & +[Q_D]^T[A_p][Q_D] \end{aligned} \tag{4.4}$$

Equation (4.4) is dependent on real and reactive power generations and demands in a network. For the purpose of transmission loss allocation, Equation (4.4) can be modified as a function of load demands.

In order to derive the modified form of loss equation some assumptions have been made. These assumptions keep the transmission loss allocation problem manageable and at the same time do not affect the results significantly. They are:

1. Constant Z-bus over the range of load variation. Although a Z-bus changes with a change in the setting of tap-changing transformers, its overall effect can be neglected.

2. Constant Bus angles for a part of the load variation. In this case, three different sets of angles have been used for the entire load range. More sets may be used for better accuracy.
3. For the first step of the iteration, the loss has been assumed to be zero. This makes the required generation equal to the load at the beginning.
4. Constant reactive to real power ratio. This reduces the number of variables in transmission loss equation.
5. Bus voltages constant at 1 p.u. for the entire range of the load.

4.4.1 Simplified Loss Equation

For a system with N buses with Kth bus taken as the reference:

$$P_G = \begin{bmatrix} P_{G1} \\ P_{G2} \\ . \\ . \\ P_{GN} \end{bmatrix}, \quad Q_G = \begin{bmatrix} Q_{G1} \\ Q_{G2} \\ . \\ . \\ Q_{GN} \end{bmatrix}, \quad P_D = \begin{bmatrix} P_{D1} \\ P_{D2} \\ . \\ . \\ P_{DN} \end{bmatrix}, \quad Q_D = \begin{bmatrix} Q_{D1} \\ Q_{D2} \\ . \\ . \\ Q_{DN} \end{bmatrix}$$

$$A_p = \begin{bmatrix} a_{p11} & a_{p12} & . & . & a_{p1N} \\ a_{p21} & a_{p22} & . & . & a_{p2N} \\ . & . & . & . & . \\ . & . & . & . & . \\ a_{pN1} & a_{pN2} & . & . & a_{pNN} \end{bmatrix}$$

$$B_p = \begin{bmatrix} b_{p11} & b_{p12} & . & . & b_{p1N} \\ b_{p21} & b_{p22} & . & . & b_{p2N} \\ . & . & . & . & . \\ . & . & . & . & . \\ b_{pN1} & b_{pN2} & . & . & b_{pNN} \end{bmatrix}$$

From the definition of A_p and B_p, the following relations can be realized,

$$a_{pij} = a_{pji}$$
$$b_{pij} = -b_{pji}$$
$$b_{pii} = 0$$

The diagonal elements of B_p are zero because of the zero angles of the sine terms in the expression of b_{pij}. The subscripts used in the matrices are in coordination with the bus numbers. Every element of Equation (4.4) is a matrix and there are sixteen terms in the equation and each term consists of three elements. It would be a huge equation obviously. Each term has been expanded separately and finally they are gathered in one single equation.

The first term of Equation (4.4) can be expanded as follows:

$$[P_G]^T[A_p][P_G] = [P_{G1} \quad P_{G2} \quad \ldots \quad P_{GN}] \begin{bmatrix} a_{p11} & a_{p12} & . & . & a_{p1N} \\ a_{p21} & a_{p22} & . & . & a_{p2N} \\ . & . & . & . & . \\ . & . & . & . & . \\ a_{pN1} & a_{pN2} & . & . & a_{pNN} \end{bmatrix} \begin{bmatrix} P_{G1} \\ P_{G2} \\ . \\ . \\ P_{GN} \end{bmatrix}$$

$$= [P_{G1} \quad P_{G2} \quad \ldots \quad P_{GN}] \begin{bmatrix} P_{g1}a_{p11} + P_{G2}a_{p12} + \ldots + P_{GN}a_{p1N} \\ P_{g1}a_{p21} + P_{G2}a_{p22} + \ldots + P_{GN}a_{p2N} \\ . \\ . \\ P_{g1}a_{pN1} + P_{G2}a_{pN2} + \ldots + P_{GN}a_{pNN} \end{bmatrix}$$

$$= P_{G1}(P_{g1}a_{p11} + P_{G2}a_{p12} + \ldots + P_{GN}a_{p1N}) + P_{G2}(P_{g1}a_{p11} + P_{G2}a_{p12} + \ldots + P_{GN}a_{p1N}) + \ldots + P_{GN}(P_{g1}a_{p11} + P_{G2}a_{p12} + \ldots + P_{GN}a_{p1N})$$

$$= \sum_{\substack{i=1 \\ i \neq K}}^{N} P_{Gi} \sum_{\substack{j=1 \\ j \neq K}}^{N} P_{Gj} a_{pij} \tag{4.5a}$$

The other terms of Equation (4.4) can be written similarly,

$$[P_D]^T[A_p][P_G] = \sum_{\substack{i=1 \\ i \neq K}}^{N} P_{Di} \sum_{\substack{j=1 \\ j \neq K}}^{N} P_{Gj} a_{pij} \tag{4.5b}$$

$$[P_G]^T[A_p][P_D] = \sum_{\substack{i=1 \\ i \neq K}}^{N} P_{Gi} \sum_{\substack{j=1 \\ j \neq K}}^{N} P_{D} a_{pij} \tag{4.5c}$$

$$[P_D]^T[A_p][P_D] = \sum_{\substack{i=1 \\ i \neq K}}^{N} P_{Di} \sum_{\substack{j=1 \\ j \neq K}}^{N} P_{Dj} a_{pij} \tag{4.5d}$$

$$[P_G]^T[B_p][Q_G] = \sum_{\substack{i=1 \\ i \neq K}}^{N} P_{Gi} \sum_{\substack{j=1 \\ j \neq K}}^{N} Q_{Gj} b_{pij} \tag{4.5e}$$

$$[P_D]^T[B_p][Q_G] = \sum_{\substack{i=1 \\ i \neq K}}^{N} P_{Di} \sum_{\substack{j=1 \\ j \neq K}}^{N} Q_{Gj} b_{pij} \tag{4.5f}$$

$$[P_G]^T[B_p][Q_D] = \sum_{\substack{i=1 \\ i \neq K}}^{N} P_{Gi} \sum_{\substack{j=1 \\ j \neq K}}^{N} Q_{Dj} b_{pij} \tag{4.5g}$$

$$[P_D]^T[B_p][Q_D] = \sum_{\substack{i=1 \\ i \neq K}}^{N} P_{Di} \sum_{\substack{j=1 \\ j \neq K}}^{N} Q_{Dj} b_{pij} \tag{4.5h}$$

$$[Q_G]^T[B_p][P_G] = \sum_{\substack{i=1 \\ i \neq K}}^{N} Q_{Gi} \sum_{\substack{j=1 \\ j \neq K}}^{N} P_{Gj} b_{pij} \tag{4.5i}$$

$$[Q_D]^T[B_p][P_G] = \sum_{\substack{i=1 \\ i \neq K}}^{N} Q_{Di} \sum_{\substack{j=1 \\ j \neq K}}^{N} P_{Gj} b_{pij} \tag{4.5j}$$

$$[Q_G]^T[B_p][P_D] = \sum_{\substack{i=1 \\ i \neq K}}^{N} Q_{Gi} \sum_{\substack{j=1 \\ j \neq K}}^{N} P_{Dj} b_{pij} \tag{4.5k}$$

$$[Q_D]^T[B_p][P_D] = \sum_{\substack{i=1 \\ i \neq K}}^{N} Q_{Di} \sum_{\substack{j=1 \\ j \neq K}}^{N} P_{Dj} b_{pij} \tag{4.5l}$$

$$[Q_G]^T[A_p][Q_G] = \sum_{\substack{i=1 \\ i \neq K}}^{N} Q_{Gi} \sum_{\substack{j=1 \\ j \neq K}}^{N} Q_{Gj} a_{pij} \tag{4.5m}$$

$$[Q_D]^T[A_p][Q_G] = \sum_{\substack{i=1 \\ i \neq K}}^{N} Q_{Di} \sum_{\substack{j=1 \\ j \neq K}}^{N} Q_{Gj} a_{pij} \tag{4.5n}$$

$$[Q_G]^T[A_p][Q_D] = \sum_{\substack{i=1\\i\neq K}}^{N} Q_{Gi} \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj} a_{pij} \tag{4.5o}$$

$$[Q_D]^T[A_p][Q_D] = \sum_{\substack{i=1\\i\neq K}}^{N} Q_{Di} \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj} a_{pij} \tag{4.5p}$$

Where,

P_{Gi} = real power generation at Bus i

P_{Di} = real load demand at Bus i

Q_{Gi} = reactive power generation at Bus i

Q_{Di} = reactive load demand at Bus i

The total transmission loss in the full system can be written, by adding Equations (4.5a)-(4.5p), as follows:

$$\begin{aligned} P_l = & \sum_{\substack{i=1\\i\neq K}}^{N} P_{Gi} \sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj} a_{pij} - \sum_{\substack{i=1\\i\neq K}}^{N} P_{Di} \sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj} a_{pij} - \sum_{\substack{i=1\\i\neq K}}^{N} P_{Gi} \sum_{\substack{j=1\\j\neq K}}^{N} P_{D} a_{pij} + \sum_{\substack{i=1\\i\neq K}}^{N} P_{Di} \sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj} a_{pij} - \sum_{\substack{i=1\\i\neq K}}^{N} P_{Gi} \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj} b_{pij} \\ & + \sum_{\substack{i=1\\i\neq K}}^{N} P_{Di} \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj} b_{pij} + \sum_{\substack{i=1\\i\neq K}}^{N} P_{Gi} \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj} b_{pij} - \sum_{\substack{i=1\\i\neq K}}^{N} P_{Di} \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj} b_{pij} + \sum_{\substack{i=1\\i\neq K}}^{N} Q_{Gi} \sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj} b_{pij} - \sum_{\substack{i=1\\i\neq K}}^{N} Q_{Di} \sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj} b_{pij} \\ & - \sum_{\substack{i=1\\i\neq K}}^{N} Q_{Gi} \sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj} b_{pij} + \sum_{\substack{i=1\\i\neq K}}^{N} Q_{Di} \sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj} b_{pij} + \sum_{\substack{i=1\\i\neq K}}^{N} Q_{Gi} \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj} a_{pij} - \sum_{\substack{i=1\\i\neq K}}^{N} Q_{Di} \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj} a_{pij} \\ & - \sum_{\substack{i=1\\i\neq K}}^{N} Q_{Gi} \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj} a_{pij} + \sum_{\substack{i=1\\i\neq K}}^{N} Q_{Di} \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj} a_{pij} \end{aligned} \tag{4.6}$$

From assumption number (4), the reactive load can be expressed in terms of real load which eventually reduces the number of variables.

$Q_{Di} = \mu \times P_{Di}$

where, μ = reactive ratio

The transmission loss equation has to be separated into respective parts for the purpose of allocating transmission loss to respective generators involved in bilateral contracts.

It is necessary to express the generation in terms of load demand and associated transmission loss. Generation in terms of loads and associated transmission losses can be expresses as:

$$P_{Gi} = \sum_{\substack{m=1 \\ m \neq K}}^{N} (P_{Dim} + L_{im}) \tag{4.7}$$

$$Q_{Gi} = \sum_{\substack{m=1 \\ m \neq K}}^{N} (\mu_{im} P_{Dim} + T_{im}) \tag{4.8}$$

$$P_{Di} = \sum_{\substack{m=1 \\ m \neq K}}^{N} (P_{Dmi}) \tag{4.9}$$

$$Q_{Di} = \sum_{\substack{m=1 \\ m \neq K}}^{N} (\mu_{mi} P_{Dmi}) \tag{4.10}$$

Where,

P_{Dim} = real load demand at Bus m which is supplied by generator connected at Bus i

L_{im} = real transmission loss allocation for the load demand at Bus m which is supplied by generator connected at Bus i

T_{im} = reactive transmission loss allocation for the load demand at Bus m which is supplied by generator connected at Bus i

μ_{im} = reactive ratio for the load at Bus m which is supplied by generator connected at Bus i

The term, P_{Dij} defines the load at Bus j that is supplied by the generator connected at Bus i. For example, the term P_{D15} is the load at Bus 5 supplied by Generator 1 and

$\sum_j P_{D1j}$ expresses the total load supplied by Generator 1 to all buses. Similarly, $\sum_j P_{Dj5}$ is the total load at Bus 5 supplied by all generators.

The expression for Q_l is basically same as the expression for P_l except that the r's in A_p and B_p are replaced by x's for the expression for Q_l. The transmission loss the share of transmission loss of any particular load can be obtained by determining the change in transmission loss due to change in the specified load.

Equation(4.5a) can be written as

$$[P_G]^T[A_p][P_G]=\sum_{\substack{i=1\\i\neq K}}^{N}\left[\sum_{\substack{m=1\\m\neq K}}^{N}(P_{Dim}+L_{im})\left\{\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}(P_{Djm}+L_{jm})\right\}\right]a_{pij} \tag{4.11}$$

Equation (4.11) can be re-written by expanding the summation terms as

$$\begin{aligned}
[P_G]^T[A_p][P_G]=&\{(P_{D11}+L_{11})+(P_{D12}+L_{12})+\ldots+(P_{D1N}+L_{1N})\}[\{(P_{D11}+L_{11})+(P_{D12}+L_{12})+\ldots\\
&+(P_{D1N}+L_{1N})\}a_{p11}+\{(P_{D21}+L_{21})+(P_{D22}+L_{22})+\ldots+(P_{D2N}+L_{2N})\}a_{p12}+\ldots\\
&\{(P_{DN1}+L_{N1})+(P_{DN2}+L_{N22})+\ldots+(P_{DNN}+L_{NN})\}a_{p1N}+\\
&\{(P_{D21}+L_{21})+(P_{D22}+L_{22})+\ldots+(P_{D2N}+L_{2N})\}[\{(P_{D11}+L_{11})+(P_{D12}+L_{12})+\ldots\\
&+(P_{D1N}+L_{1N})\}a_{p11}+\{(P_{D21}+L_{21})+(P_{D22}+L_{22})+\ldots+(P_{D2N}+L_{2N})\}a_{p12}+\ldots\\
&\{(P_{DN1}+L_{N1})+(P_{DN2}+L_{N22})+\ldots+(P_{DNN}+L_{NN})\}a_{p1N}+\ldots\\
&\{(P_{DN1}+L_{N1})+(P_{DN2}+L_{N2})+\ldots+(P_{DNN}+L_{NN})\}[\{(P_{D11}+L_{11})+(P_{D12}+L_{12})+\ldots\\
&+(P_{D1N}+L_{1N})\}a_{p11}+\{(P_{D21}+L_{21})+(P_{D22}+L_{22})+\ldots+(P_{D2N}+L_{2N})\}a_{p12}+\ldots\\
&\{(P_{DN1}+L_{N1})+(P_{DN2}+L_{N22})+\ldots+(P_{DNN}+L_{NN})\}a_{p1N}
\end{aligned} \tag{4.12a}$$

Equation (4.12a) represents the expanded form of the first term of Equation (4.6). It can be differentiated with respect to load P_{D11} in the following manner.

$$\frac{\partial[P_G]^T[A_p][P_G]}{\partial P_{D11}}=\left(1+\frac{\partial L_{11}}{\partial P_{D11}}\right)\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\left(P_{Djm}+L_{jm}\right)a_{p1j}+\sum_{\substack{m=1\\m\neq K}}^{N}\left(P_{D1m}+L_{1m}\right)\left[\left(1+\frac{\partial L_{11}}{\partial P_{D11}}\right)a_{p11}\right]+$$

$$\sum_{\substack{m=1\\m\neq K}}^{N}\left(P_{D2m}+L_{2m}\right)\left[\left(1+\frac{\partial L_{11}}{\partial P_{D11}}\right)a_{p21}\right]+\ldots$$

$$=\left(1+U_{11}\right)\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\left(P_{Djm}+L_{jm}\right)a_{p1j}+\left(1+U_{11}\right)\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\left(P_{Djm}+L_{jm}\right)a_{pj1}$$

(4.13a)

Similarly, differentiating with respect to load P_{D12}, P_{D21} etc. we get

$$\frac{\partial[P_G]^T[A_p][P_G]}{\partial P_{D12}}=\left(1+\frac{\partial L_{12}}{\partial P_{D12}}\right)\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\left(P_{Djm}+L_{jm}\right)a_{p1j}+\sum_{\substack{m=1\\m\neq K}}^{N}\left(P_{D1m}+L_{1m}\right)\left[\left(1+\frac{\partial L_{12}}{\partial P_{D12}}\right)a_{p11}\right]+$$

$$\sum_{\substack{m=1\\m\neq K}}^{N}\left(P_{D2m}+L_{2m}\right)\left[\left(1+\frac{\partial L_{121}}{\partial P_{D12}}\right)a_{p21}\right]+\ldots$$

$$=\left(1+U_{12}\right)\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\left(P_{Djm}+L_{jm}\right)a_{p1j}+\left(1+U_{12}\right)\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\left(P_{Djm}+L_{jm}\right)a_{pj1}$$

$$\frac{\partial[P_G]^T[A_p][P_G]}{\partial P_{D21}}=\left(1+\frac{\partial L_{21}}{\partial P_{D21}}\right)\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\left(P_{Djm}+L_{jm}\right)a_{p2j}+\sum_{\substack{m=1\\m\neq K}}^{N}\left(P_{D1m}+L_{1m}\right)\left[\left(1+\frac{\partial L_{21}}{\partial P_{D21}}\right)a_{p12}\right]+$$

$$\sum_{\substack{m=1\\m\neq K}}^{N}\left(P_{D2m}+L_{2m}\right)\left[\left(1+\frac{\partial L_{21}}{\partial P_{D21}}\right)a_{p22}\right]+\ldots$$

$$=\left(1+U_{21}\right)\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\left(P_{Djm}+L_{jm}\right)a_{p2j}+\left(1+U_{21}\right)\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\left(P_{Djm}+L_{jm}\right)a_{pj2}$$

(4.13b)

In general, Equation (4.12a) can be differentiated with respect to load, P_{Dxy},

$$\frac{\partial [P_G]^T [A_p][P_G]}{\partial P_{Dxy}} = (1+U_{xy})\sum_{\substack{j=1\\ j\neq K}}^{N}\sum_{\substack{m=1\\ m\neq K}}^{N}(P_{Djm}+L_{jm})a_{pxj} + (1+U_{xy})\sum_{\substack{j=1\\ j\neq K}}^{N}\sum_{\substack{m=1\\ m\neq K}}^{N}(P_{Djm}+L_{jm})a_{pjx} \tag{4.14a}$$

Similar operations can be performed on the rest of the terms of Equation (4.6) and organized in the same form as Equation (4.14a). Individual terms and their derivatives with respect to P_{Dxy} are shown below:

$$[P_D]^T [A_p][P_G] = \sum_{\substack{i=1\\ i\neq K}}^{N}\left[\sum_{\substack{m=1\\ m\neq K}}^{N} P_{Dmi}\left\{\sum_{\substack{j=1\\ j\neq K}}^{N}\sum_{\substack{m=1\\ m\neq K}}^{N}(P_{Djm}+L_{jm})\right\}\right] a_{pij} \tag{4.12b}$$

$$\frac{\partial [P_D]^T [A_p][P_G]}{\partial P_{Dxy}} = (1+U_{xy})\sum_{\substack{j=1\\ j\neq K}}^{N}\sum_{\substack{m=1\\ m\neq K}}^{N} P_{Dmj}\,a_{pjx} + \sum_{\substack{j=1\\ j\neq K}}^{N}\sum_{\substack{m=1\\ m\neq K}}^{N}(P_{Djm}+L_{jm})a_{pyj} \tag{4.14b}$$

$$[P_G]^T [A_p][P_D] = \sum_{\substack{i=1\\ i\neq K}}^{N}\left[\sum_{\substack{m=1\\ m\neq K}}^{N}(P_{Dim}+L_{im})\left\{\sum_{\substack{j=1\\ j\neq K}}^{N}\sum_{\substack{m=1\\ m\neq K}}^{N} P_{Dmj}\right\}\right] a_{pij} \tag{4.12c}$$

$$\frac{\partial [P_G]^T [A_p][P_D]}{\partial P_{Dxy}} = (1+U_{xy})\sum_{\substack{j=1\\ j\neq K}}^{N}\sum_{\substack{m=1\\ m\neq K}}^{N} P_{Dmj}\,a_{xj} + \sum_{\substack{j=1\\ j\neq K}}^{N}\sum_{\substack{m=1\\ m\neq K}}^{N}(P_{Djm}+L_{jm})a_{pjy} \tag{4.14c}$$

$$[P_D]^T [A_p][P_D] = \sum_{\substack{i=1\\ i\neq K}}^{N}\left[\sum_{\substack{m=1\\ m\neq K}}^{N} P_{Dmi}\left\{\sum_{\substack{j=1\\ j\neq K}}^{N}\sum_{\substack{m=1\\ m\neq K}}^{N} P_{Dmj}\right\}\right] a_{pij} \tag{4.12d}$$

$$\frac{\partial [P_D]^T [A_p][P_D]}{\partial P_{Dxy}} = \sum_{\substack{j=1\\ j\neq K}}^{N}\sum_{\substack{m=1\\ m\neq K}}^{N} P_{Dmj}\,a_{pyj} + \sum_{\substack{j=1\\ j\neq K}}^{N}\sum_{\substack{m=1\\ m\neq K}}^{N} P_{Dmj}\,a_{pjy} \tag{4.14d}$$

$$[P_G]^T [B_p][Q_G] = \sum_{\substack{i=1\\ i\neq K}}^{N}\left[\sum_{\substack{m=1\\ m\neq K}}^{N}(P_{Dim}+L_{im})\left\{\sum_{\substack{j=1\\ j\neq K}}^{N}\sum_{\substack{m=1\\ m\neq K}}^{N}(\mu_{jm}P_{Djm}+T_{jm})\right\}\right] b_{pij} \tag{4.12e}$$

$$\frac{\partial[P_G]^T[B_p][Q_G]}{\partial P_{Dxy}}=\left(1+U_{xy}\right)\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\left(\mu_{xy}P_{Djm}+T_{jm}\right)b_{pxj}+\left(\mu_{xy}+W_{xy}\right)\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\left(P_{Djm}+L_{jm}\right)b_{pjx} \tag{4.14e}$$

$$[P_D]^T[B_p][Q_G]=\sum_{\substack{i=1\\i\neq K}}^{N}\left[\sum_{\substack{m=1\\m\neq K}}^{N}P_{Dmi}\left\{\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\left(\mu_{jm}P_{Djm}+T_{jm}\right)\right\}\right]b_{pij} \tag{4.12f}$$

$$\frac{\partial[P_D]^T[B_p][Q_G]}{\partial P_{Dxy}}=\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\left(\mu_{jm}P_{Djm}+T_{jm}\right)b_{pyj}+\left(\mu_{xy}+W_{xy}\right)\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}P_{Dmj}\,b_{pjx} \tag{4.14f}$$

$$[P_G]^T[B_p][Q_D]=\sum_{\substack{i=1\\i\neq K}}^{N}\left[\sum_{\substack{m=1\\m\neq K}}^{N}\left(P_{Dim}+L_{im}\right)\left\{\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\mu_{mj}P_{Dmj}\right\}\right]b_{pij} \tag{4.12g}$$

$$\frac{\partial[P_G]^T[B_p][Q_D]}{\partial P_{Dxy}}=\left(1+U_{xy}\right)\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\mu_{mj}P_{Dmj}b_{pxj}+\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\left(P_{Djm}+L_{jm}\right)b_{pjy} \tag{4.14g}$$

$$[P_D]^T[B_p][Q_D]=\sum_{\substack{i=1\\i\neq K}}^{N}\left[\sum_{\substack{m=1\\m\neq K}}^{N}P_{Dmi}\left\{\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\mu_{mj}P_{Dmj}\right\}\right]b_{pij} \tag{4.12h}$$

$$\frac{\partial[P_D]^T[B_p][Q_D]}{\partial P_{Dxy}}=\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\mu_{mj}P_{Dmj}b_{pyj}+\mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}P_{Dmj}\,b_{pjy} \tag{4.14h}$$

$$[Q_G]^T[B_p][P_G]=\sum_{\substack{i=1\\i\neq K}}^{N}\left[\sum_{\substack{m=1\\m\neq K}}^{N}\left(\mu_{im}P_{Dim}+T_{im}\right)\left\{\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\left(P_{Djm}+L_{jm}\right)\right\}\right]b_{pij} \tag{4.12i}$$

$$\frac{\partial[Q_G]^T[B_p][P_G]}{\partial P_{Dxy}}=\left(\mu_{xy}+W_{xy}\right)\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\left(P_{Djm}+L_{jm}\right)b_{pxj}+\left(1+U_{xy}\right)\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\left(\mu_{jm}P_{Djm}+T_{jm}\right)b_{pjx} \tag{4.14i}$$

$$[Q_D]^T[B_p][P_G]=\sum_{\substack{i=1\\i\neq K}}^{N}\left[\sum_{\substack{m=1\\m\neq K}}^{N}\mu_{mi}P_{Dmi}\left\{\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\left(P_{Djm}+L_{jm}\right)\right\}\right]b_{pij} \tag{4.12j}$$

$$\frac{\partial[Q_D]^T[B_p][P_G]}{\partial P_{Dxy}}=\mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\left(P_{Djm}+L_{jm}\right)b_{pyj}+\left(1+U_{xy}\right)\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\mu_{mj}P_{Dmj}b_{pjx} \tag{4.14j}$$

$$[Q_G]^T[B_p][P_D]=\sum_{\substack{i=1\\i\neq K}}^{N}\left[\sum_{\substack{m=1\\m\neq K}}^{N}\left(\mu_{im}P_{Dim}+P_{Dim}\right)\left\{\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}P_{Dmj}\right\}\right]b_{pij} \tag{4.12k}$$

$$\frac{\partial[Q_G]^T[B_p][P_D]}{\partial P_{Dxy}}=\left(\mu_{xy}+W_{xy}\right)\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}P_{Dmj}b_{pxj}+\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\left(\mu_{jm}P_{Djm}+T_{jm}\right)b_{pjy} \tag{4.14k}$$

$$[Q_D]^T[B_p][P_D]=\sum_{\substack{i=1\\i\neq K}}^{N}\left[\sum_{\substack{m=1\\m\neq K}}^{N}\mu_{mi}P_{Dmi}\left\{\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}P_{Dmj}\right\}\right]b_{pij} \tag{4.12l}$$

$$\frac{\partial[Q_D]^T[B_p][P_D]}{\partial P_{Dxy}}=\mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}P_{Dmj}b_{pyj}+\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\mu_{mj}P_{Dmj}b_{pjy} \tag{4.14l}$$

$$[Q_G]^T[A_p][Q_G]=\sum_{\substack{i=1\\i\neq K}}^{N}\left[\sum_{\substack{m=1\\m\neq K}}^{N}\left(\mu_{im}P_{Dim}+T_{im}\right)\left\{\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\left(\mu_{jm}P_{Djm}+T_{jm}\right)\right\}\right]a_{pij} \tag{4.12m}$$

$$\frac{\partial[Q_G]^T[A_p][Q_G]}{\partial P_{Dxy}}=\left(\mu_{xy}+W_{xy}\right)\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\left(\mu_{jm}P_{Djm}+T_{jm}\right)a_{pxj}+\left(\mu_{xy}+W_{xy}\right)\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\left(\mu_{jm}P_{Djm}+T_{jm}\right)a_{pjx} \tag{4.14m}$$

$$[Q_D]^T[A_p][Q_G]=\sum_{\substack{i=1\\i\neq K}}^{N}\left[\sum_{\substack{m=1\\m\neq K}}^{N}\mu_{mi}P_{Dmi}\left\{\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}(\mu_{jm}P_{Djm}+T_{jm})\right\}\right]a_{pij} \tag{4.12n}$$

$$\frac{\partial[Q_D]^T[A_p][Q_G]}{\partial P_{Dxy}}=\mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}(\mu_{jm}P_{Djm}+T_{jm})a_{pyj}+(\mu_{xy}+W_{xy})\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\mu_{mj}P_{Dmj}a_{pjx} \tag{4.14n}$$

$$[Q_G]^T[A_p][Q_D]=\sum_{\substack{i=1\\i\neq K}}^{N}\left[\sum_{\substack{m=1\\m\neq K}}^{N}(\mu_{im}P_{Dim}+T_{im})\left\{\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\mu_{mj}P_{Dmj}\right\}\right]a_{pij} \tag{4.12o}$$

$$\frac{\partial[Q_G]^T[A_p][Q_D]}{\partial P_{Dxy}}=(\mu_{xy}+W_{xy})\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\mu_{mj}P_{Dmj}a_{pxj}+\mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}(\mu_{jm}P_{Djm}+T_{jm})a_{pjy} \tag{4.14o}$$

$$[Q_D]^T[A_p][Q_D]=\sum_{\substack{i=1\\i\neq K}}^{N}\left[\sum_{\substack{m=1\\m\neq K}}^{N}\mu_{mi}P_{Dmi}\left\{\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\mu_{mj}P_{Dmj}\right\}\right]a_{pij} \tag{4.12p}$$

$$\frac{\partial[Q_D]^T[A_p][Q_D]}{\partial P_{Dxy}}=\mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\mu_{mj}P_{Dmj}a_{pyj}+\mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N}\sum_{\substack{m=1\\m\neq K}}^{N}\mu_{mj}P_{Dmj}a_{pjy} \tag{4.14p}$$

Assume,

L_{xy} = share of real transmission loss of Generator x for supplying load connected at Bus y.

T_{xy} = share of reactive transmission loss of Generator x for supplying load connected at Bus y.

$$U_{xy}=\frac{\partial L_{xy}}{\partial P_{Dxy}}$$

$$W_{xy}=\frac{\partial T_{xy}}{\partial P_{Dxy}}$$

The individual terms of Equation (4.14) can be written with their appropriate signs as:

$$\frac{\partial[P_G]^T[A_p][P_G]}{\partial P_{Dxy}} = 2U_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\,a_{pxj} + 2\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\,a_{pxj} \tag{4.15a}$$

$$-\frac{\partial[P_D]^T[A_p][P_G]}{\partial P_{Dxy}} = -U_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\,a_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\,a_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\,a_{pyj} \tag{4.15b}$$

$$-\frac{\partial[P_G]^T[A_p][P_D]}{\partial P_{Dxy}} = -U_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\,a_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\,a_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\,a_{pyj} \tag{4.15c}$$

$$\frac{\partial[P_D]^T[A_p][P_D]}{\partial P_{Dxy}} = 2\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\,a_{pyj} \tag{4.15d}$$

$$-\frac{\partial[P_G]^T[B_p][Q_G]}{\partial P_{Dxy}} = -U_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj}b_{pxj} + W_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\,b_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj}b_{pxj} + \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\,b_{pxj} \tag{4.15e}$$

$$\frac{\partial[P_D]^T[B_p][Q_G]}{\partial P_{Dxy}} = -W_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\,b_{pxj} + \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj}\,b_{pyj} - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\,b_{pxj} \tag{4.15f}$$

$$\frac{\partial[P_G]^T[B_p][Q_D]}{\partial P_{Dxy}} = U_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}\,b_{pxj} + \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}\,b_{pxj} - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\,b_{pyj} \tag{4.15g}$$

$$-\frac{\partial[P_D]^T[B_p][Q_D]}{\partial P_{Dxy}} = -\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}b_{pyj} + \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\,b_{pyj} \tag{4.15h}$$

$$\frac{\partial[Q_G]^T[B_p][P_G]}{\partial P_{Dxy}} = -U_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj}b_{pxj} + W_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\,b_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj}b_{pxj} + \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\,b_{pxj} \qquad (4.15i)$$

$$-\frac{\partial[Q_D]^T[B_p][P_G]}{\partial P_{Dxy}} = U_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}\,b_{pxj} + \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}\,b_{pxj} - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\,b_{pyj} \qquad (4.15j)$$

$$-\frac{\partial[Q_G]^T[B_p][P_D]}{\partial P_{Dxy}} = -W_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\,b_{pxj} + \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj}\,b_{pyj} - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\,b_{pxj} \qquad (4.15k)$$

$$\frac{\partial[Q_D]^T[B_p][P_D]}{\partial P_{Dxy}} = \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}b_{pyj} - \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}\,b_{pyj} \qquad (4.15l)$$

$$\frac{\partial[Q_G]^T[A_p][Q_G]}{\partial P_{Dxy}} = 2W_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj}\,a_{pxj} + 2\mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj}\,a_{pxj} \qquad (4.15m)$$

$$-\frac{\partial[Q_D]^T[A_p][Q_G]}{\partial P_{Dxy}} = -W_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}\,a_{pxj} - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}\,a_{pxj} - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj}\,a_{pyj} \qquad (4.15n)$$

$$-\frac{\partial[Q_G]^T[A_p][Q_D]}{\partial P_{Dxy}} = -W_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}\,a_{pxj} - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}\,a_{pxj} - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj}\,a_{pyj} \qquad (4.15o)$$

$$\frac{\partial[Q_D]^T[A_p][Q_D]}{\partial P_{Dxy}} = 2\mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}\,a_{pyj} \qquad (4.15p)$$

The sum of all terms shown in Equation (4.15a) to Equation (4.15p) represents the differentiation of transmission loss Equation (4.6) with respect to a generalized load P_{xy}. The following equation shows the change in total transmission loss with respect to change in any load P_{xy}.

$$
\begin{aligned}
\frac{\partial P_l}{\partial P_{Dxy}} = {} & 2U_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\, a_{pxj} + 2\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\, a_{pxj} - U_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\, a_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\, a_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\, a_{pyj} \\
& - U_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\, a_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\, a_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\, a_{pyj} + 2\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\, a_{pyj} - U_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj} b_{pxj} \\
& + W_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\, b_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj} b_{pxj} + \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\, b_{pxj} - W_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\, b_{pxj} + \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj}\, b_{pyj} \\
& - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\, b_{pxj} + U_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}\, b_{pxj} + \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}\, b_{pxj} - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\, b_{pyj} - \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj} b_{pyj} \\
& + \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\, b_{pyj} - U_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj} b_{pxj} + W_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\, b_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj} b_{pxj} + \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\, b_{pxj} \\
& + U_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}\, b_{pxj} + \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}\, b_{pxj} - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\, b_{pyj} - W_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\, b_{pxj} + \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj}\, b_{pyj} \\
& - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\, b_{pxj} + \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj} b_{pyj} - \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}\, b_{pyj} + 2W_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj} a_{pxj} + 2\mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj} a_{pxj} \\
& - W_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj} a_{pxj} - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj} a_{pxj} - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj} a_{pyj} - W_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj} a_{pxj} - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj} a_{pxj} \\
& - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj} a_{pyj} + 2\mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj} a_{pyj}
\end{aligned}
\tag{4.16}
$$

4.4.2 Solution Approach

In a deregulated environment, a generator may enter into a bilateral contract with a load. Since, transmission loss does not vary linearly and depends on the network configuration and relative position of the generators and loads, it is difficult for a Generator to know how much power to produce for supplying its bilaterally contracted load. Therefore, it is necessary to break up the total transmission loss so that each generator would know its share of transmission for supplying a specified load in the system. The total transmission loss can be written as:

$$P_l = \sum_{\substack{i=1 \\ 1 \neq K}}^{N} \sum_{\substack{i=1 \\ 1 \neq K}}^{N} L_{ij} \tag{4.17}$$

$$Q_l = \sum_{\substack{i=1 \\ 1 \neq K}}^{N} \sum_{\substack{i=1 \\ 1 \neq K}}^{N} T_{ij} \tag{4.18}$$

Where,

L_{ij} = share of real transmission loss of Generator i for supplying the load connected at Bus j.

T_{ij} = share of reactive transmission loss of Generator i for supplying the load connected at Bus j.

By utilizing MTLA, a load can be increased from zero to the respective customer's demand in a successive process and the corresponding change in the transmission loss for each load increment can be expressed as:

$$\begin{aligned}
\Delta P_l &= \Delta L_{11} + \ldots + \Delta L_{21} + \Delta L_{22} + \ldots + \Delta L_{xy} + \ldots + \Delta L_{NN} \\
&= \frac{\partial P_l}{\partial P_{D11}} \Delta P_{D11} + \frac{\partial P_l}{\partial P_{D12}} \Delta P_{D12} + \ldots + \frac{\partial P_l}{\partial P_{Dxy}} \Delta P_{Dxy} + \ldots + \frac{\partial P_l}{\partial P_{DNN}} \Delta P_{DNN} \\
&= U_{11} \Delta P_{D11} + U_{12} \Delta P_{D12} + \ldots + U_{xy} \Delta P_{Dxy} + \ldots + U_{NN} \Delta P_{DNN}
\end{aligned} \tag{4.19}$$

$$\begin{aligned}
\Delta Q_l &= \Delta T_{11} + \ldots + \Delta T_{21} + \Delta T_{22} + \ldots + \Delta T_{xy} + \ldots + \Delta T_{NN} \\
&= \frac{\partial Q_l}{\partial P_{D11}} \Delta P_{D11} + \frac{\partial Q_l}{\partial P_{D12}} \Delta P_{D12} + \ldots + \frac{\partial Q_l}{\partial P_{Dxy}} \Delta P_{Dxy} + \ldots + \frac{\partial Q_l}{\partial P_{DNN}} \Delta P_{DNN} \\
&= W_{11} \Delta P_{D11} + W_{12} \Delta P_{D12} + \ldots + W_{xy} \Delta P_{Dxy} + \ldots + W_{NN} \Delta P_{DNN}
\end{aligned} \tag{4.20}$$

Where,

ΔP_l = total change in real transmission loss.

ΔQ_l = total change in reactive transmission loss.

ΔL_{xy} = change in share of real transmission loss of the Generator connected at Bus x for supplying the load connected at Bus y.

ΔT_{xy} = change in share of real transmission loss of the Generator connected at Bus x for supplying the load connected at Bus y.

Change in individual real transmission loss can be written as

$$\Delta L_{xy} = \frac{\partial P_l}{\partial P_{Dxy}} \Delta P_{Dxy}$$

$$\text{or, } \frac{\partial P_l}{\partial P_{Dxy}} = \frac{\Delta L_{xy}}{\Delta P_{Dxy}}$$

If ΔL_{xy} and ΔP_{Dxy} are small then,

$$\frac{\partial P_l}{\partial P_{Dxy}} \approx \frac{\partial L_{xy}}{\partial P_{Dxy}} = U_{xy}$$

Similarly for individual reactive transmission loss, change can be expressed as:

$$\frac{\partial Q_l}{\partial P_{Dxy}} \approx \frac{\partial T_{xy}}{\partial P_{Dxy}} = W_{xy}$$

The total transmission loss in a power system network is given by Equation (4.6). In this equation, the generation terms (P_Gs and Q_Gs) are replaceable by Equations (4.7) and (4.8). In Equation (4.6), generation terms are used in order to keep the equation size manageable.

$$\begin{aligned}
P_l = & \sum_{\substack{i=1\\i\neq K}}^{N} P_{Gi} \sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj} a_{pij} - \sum_{\substack{i=1\\i\neq K}}^{N} P_{Di} \sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj} a_{pij} - \sum_{\substack{i=1\\i\neq K}}^{N} P_{Gi} \sum_{\substack{j=1\\j\neq K}}^{N} P_{D} a_{pij} + \sum_{\substack{i=1\\i\neq K}}^{N} P_{Di} \sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj} a_{pij} - \\
& \sum_{\substack{i=1\\i\neq K}}^{N} P_{Gi} \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj} b_{pij} + \sum_{\substack{i=1\\i\neq K}}^{N} P_{Di} \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj} b_{pij} + \sum_{\substack{i=1\\i\neq K}}^{N} P_{Gi} \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj} b_{pij} - \sum_{\substack{i=1\\i\neq K}}^{N} P_{Di} \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj} b_{pij} + \\
& \sum_{\substack{i=1\\i\neq K}}^{N} Q_{Gi} \sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj} b_{pij} - \sum_{\substack{i=1\\i\neq K}}^{N} Q_{Di} \sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj} b_{pij} - \sum_{\substack{i=1\\i\neq K}}^{N} Q_{Gi} \sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj} b_{pij} + \sum_{\substack{i=1\\i\neq K}}^{N} Q_{Di} \sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj} b_{pij} + \\
& \sum_{\substack{i=1\\i\neq K}}^{N} Q_{Gi} \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj} a_{pij} - \sum_{\substack{i=1\\i\neq K}}^{N} Q_{Di} \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj} a_{pij} - \sum_{\substack{i=1\\i\neq K}}^{N} Q_{Gi} \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj} a_{pij} + \sum_{\substack{i=1\\i\neq K}}^{N} Q_{Di} \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj} a_{pij}
\end{aligned} \tag{4.21}$$

Differentiating the transmission loss expression P_l, Equation (4.21) with respect to P_{Dxy} we get,

$$
\begin{aligned}
\frac{\partial P_l}{\partial P_{Dxy}} = {} & 2U_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\,a_{pxj} + 2\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\,a_{pxj} - U_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\,a_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\,a_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\,a_{pyj} - \\
& U_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\,a_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\,a_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\,a_{pyj} + 2\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\,a_{pyj} - U_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj} b_{pxj} + \\
& W_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\,b_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj} b_{pxj} + \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\,b_{pxj} - W_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\,b_{pxj} + \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj}\,b_{pyj} - \\
& \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\,b_{pxj} + U_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}\,b_{pxj} + \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}\,b_{pxj} - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\,b_{pyj} - \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj} b_{pyj} + \\
& \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\,b_{pyj} - U_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj} b_{pxj} + W_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\,b_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj} b_{pxj} + \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\,b_{pxj} + \\
& U_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}\,b_{pxj} + \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}\,b_{pxj} - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}\,b_{pyj} - W_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\,b_{pxj} + \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj}\,b_{pyj} - \\
& \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}\,b_{pxj} + \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj} b_{pyj} - \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}\,b_{pyj} + 2W_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj} a_{pxj} + 2\mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj} a_{pxj} - \\
& W_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj} a_{pxj} - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj} a_{pxj} - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj} a_{pyj} - W_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj} a_{pxj} - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj} a_{pxj} - \\
& \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj} a_{pyj} + 2\mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj} a_{pyj}
\end{aligned}
\tag{4.22}
$$

After reorganizing, Equation (4.22) can be written as:

$$\alpha_{xy} U_{xy} + \beta_{xy} W_{xy} = \gamma_{xy} \tag{4.23}$$

where,

$$\alpha_{xy} = 1 - 2\left\{ \sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj} a_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj} a_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj} b_{pxj} + \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj} a_{pxj} \right\}$$

$$\beta_{xy} = -2\left\{ \sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj} b_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj} b_{pxj} + \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj} a_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj} a_{pxj} \right\}$$

$$\gamma_{xy} = 2\left\{\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}a_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}a_{pyj} - \sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}a_{pxj} + \sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}a_{pyj} - \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj}b_{pxj} + \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}b_{pxj}\right.$$

$$+ \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj}b_{pyj} - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}b_{pxj} + \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}b_{pxj} - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}b_{pyj} + \mu_{xy}\sum_{\substack{j=1\\j\neq k}}^{N} P_{Dj}b_{pyj} -$$

$$\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}b_{pyj} + \mu_{xy}\sum_{\substack{j=1\\j\neq k}}^{N} Q_{Gj}a_{pxj} - \mu_{xy}\sum_{\substack{j=1\\j\neq k}}^{N} Q_{Gj}a_{pyj} - \mu_{xy}\sum_{\substack{j=1\\j\neq k}}^{N} Q_{Dj}a_{pxj} +$$

$$\left.\mu_{xy}\sum_{\substack{j=1\\j\neq k}}^{N} Q_{Dj}a_{pyj}\right\}$$

Now a second equation is required for the solution of U_{xy} and W_{xy} which would be obtained from Equation (4.21). By examining Equations (2.9) and (2.10), it becomes clear that expression for Q_l will be similar to that of P_l. With the exceptions of a_{pij} and b_{pij} elements where r's will be replaced by x's. The expression for Q_l can be written as:

$$Q_l = \sum_{\substack{i=1\\i\neq K}}^{N} P_{Gi}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}a_{pij} - \sum_{\substack{i=1\\i\neq K}}^{N} P_{Di}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}a_{pij} - \sum_{\substack{i=1\\i\neq K}}^{N} P_{Gi}\sum_{\substack{j=1\\j\neq K}}^{N} P_{D}a_{pij} + \sum_{\substack{i=1\\i\neq K}}^{N} P_{Di}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}a_{pij} -$$

$$\sum_{\substack{i=1\\i\neq K}}^{N} P_{Gi}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj}b_{pij} + \sum_{\substack{i=1\\i\neq K}}^{N} P_{Di}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj}b_{pij} + \sum_{\substack{i=1\\i\neq K}}^{N} P_{Gi}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}b_{pij} - \sum_{\substack{i=1\\i\neq K}}^{N} P_{Di}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}b_{pij} +$$

$$\sum_{\substack{i=1\\i\neq K}}^{N} Q_{Gi}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}b_{pij} - \sum_{\substack{i=1\\i\neq K}}^{N} Q_{Di}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}b_{pij} - \sum_{\substack{i=1\\i\neq K}}^{N} Q_{Gi}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}b_{pij} + \sum_{\substack{i=1\\i\neq K}}^{N} Q_{Di}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}b_{pij} +$$

$$\sum_{\substack{i=1\\i\neq K}}^{N} Q_{Gi}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj}a_{pij} - \sum_{\substack{i=1\\i\neq K}}^{N} Q_{Di}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj}a_{pij} - \sum_{\substack{i=1\\i\neq K}}^{N} Q_{Gi}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}a_{pij} + \sum_{\substack{i=1\\i\neq K}}^{N} Q_{Di}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}a_{pij}$$

(4.24)

where,

$$a_{pij} = \frac{x_{ij}}{|V_i||V_j|}\cos(\delta_i - \delta_j)$$

$$b_{pij} = \frac{x_{ij}}{|V_i||V_j|}\sin(\delta_i - \delta_j) \qquad (4.25)$$

After differentiating Q_l with respect to P_{Dxy} and performing the mathematical operations similar to that done for Equation (4.21), the following can be written:

$$\alpha'_{xy}U_{xy} + \beta'_{xy}W_{xy} = \gamma'_{xy} \tag{4.26}$$

where,

$$\alpha'_{xy} = -2\left\{\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}a_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}a_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj}b_{pxj} + \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}a_{pxj}\right\}$$

$$\beta'_{xy} = 1 - 2\left\{\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}b_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}b_{pxj} + \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj}a_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}a_{pxj}\right\}$$

$$\begin{aligned}\gamma'_{xy} = 2\Bigg\{&\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}a_{pxj} - \sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}a_{pyj} - \sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}a_{pxj} + \sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}a_{pyj} - \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj}b_{pxj} + \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}b_{pxj}\\ &+ \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj}b_{pyj} - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}b_{pxj} + \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}b_{pxj} - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Gj}b_{pyj} + \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} P_{Dj}b_{pyj} - \sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}b_{pyj}\\ &+ \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj}a_{pxj} - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Gj}a_{pyj} - \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}a_{pxj} + \mu_{xy}\sum_{\substack{j=1\\j\neq K}}^{N} Q_{Dj}a_{pyj}\Bigg\}\end{aligned}$$

Solving Equations (4.23) and (4.26) simultaneously for U_{xy} and W_{xy},

$$\begin{aligned}U_{xy} &= \frac{\gamma_{xy}\beta'_{xy} - \gamma'_{xy}\beta_{xy}}{\alpha_{xy}\beta'_{xy} - \alpha'_{xy}\beta_{xy}}\\ W_{xy} &= \frac{\alpha_{xy}\gamma'_{xy} - \alpha'_{xy}\gamma_{xy}}{\alpha_{xy}\beta'_{xy} - \alpha'_{xy}\beta_{xy}}\end{aligned} \tag{4.27}$$

It is important to note that a_{pij} and b_{pij} parameters for the reactive part have to be calculated by the relationship given in Equation (4.25).

After evaluating U_{xy} and V_{xy}, the share of transmission loss of each generator can be evaluated using Equations (4.19) and (4.20).

4.5 Example System

Consider the hypothetical system shown in Figure 3.1 where Generator A, connected to Bus1, is supplying power to its contracted customer A, connected to Bus5. The real and reactive power generation by Generator A can be written mathematically as:

$$P_{G1} = P_{D15} + L_{15} \tag{4.28}$$

$$Q_{G1} = \mu_{15} P_{D15} + T_{15} \tag{4.29}$$

Similarly, the real and reactive power generation of Generator B can be written as:

$$P_{G2} = P_{D26} + L_{26} \tag{4.30}$$

$$Q_{G2} = \mu_{26} P_{D26} + T_{26} \tag{4.31}$$

The loss allocations for both generators are obtained by determining the parameters U_{15}, W_{15}, U_{26} and W_{26} and using Equations (4.19) and (4.20).

Previously, the loss allocations for Generator A and B were obtained using the equations which were derived for this particular system shown in Fig. 3.1 [48]. Those equations were not generalized and cannot be used in any other system. It has been suggested that for other networks new set of equations would be required. In this work, a generalized equation has been developed that would be applicable to any power system network. Some results are shown below in Tables 4.4-4.5 obtained by the particular set of equations [48,70] and the equations derived in this work. Different load parameters have been used for obtaining the loss allocations utilizing the newly derived generalized equation.

The demand of Customers A and B have been kept fixed and their reactive power ratio has been varied from 0.6 to 0.3. Allocations of transmission loss have been calculated for the corresponding variations in reactive power. Tables 4.4 and 4.5 show the calculated allocation of transmission loss for Generators A and B. The real load demand of Customer A and B is 1.5 p.u.

It can be noticed from Tables 4.1 and 4.2 that for both real and reactive powers, difference in loss allocation for Generator A, obtained from two different methods, increases with a decrease of reactive ratio of Customer A while the opposite happens to Generator B.

Table 4.1: Real Loss Allocation for equal load and equal reactive ratio.

Reactive Ratio	A's Share of Loss (p.u.)		B's Share of Loss (p.u.)	
μ	ILFA	MTLA	ILFA	MTLA
0.6	0.0405	0.0412	0.0579	0.0609
0.5	0.0360	0.0370	0.0518	0.0544
0.4	0.0322	0.0337	0.0469	0.0484
0.3	0.0293	0.0310	0.0432	0.0443

Table 4.2: Reactive Loss Allocation for equal load and equal reactive ratio.

Reactive Ratio	A's Share of Loss (p.u.)		B's Share of Loss (p.u.)	
μ	ILFA	MTLA	ILFA	MTLA
0.6	0.2430	0.2441	0.3277	0.3398
0.5	0.2158	0.2191	0.2934	0.3038
0.4	0.1940	0.2000	0.2653	0.2705
0.3	0.1769	0.1838	0.2432	0.2475

Figures 4.1 to 4.4 show the real and reactive loss allocation for both generators for a reactive ratio of 0.6. Each figure shows two curves, one from the MTLA and the other from the ILFA.

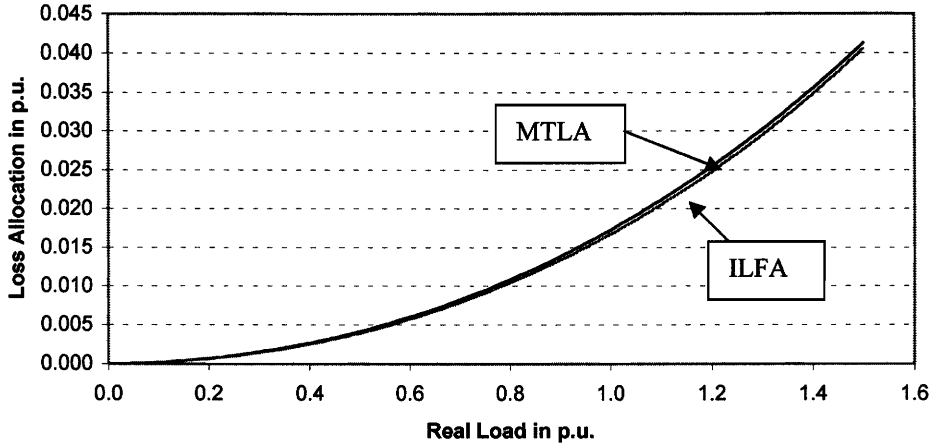

Fig. 4.1: Allocation of real transmission loss for Generator A (μ=0.6).

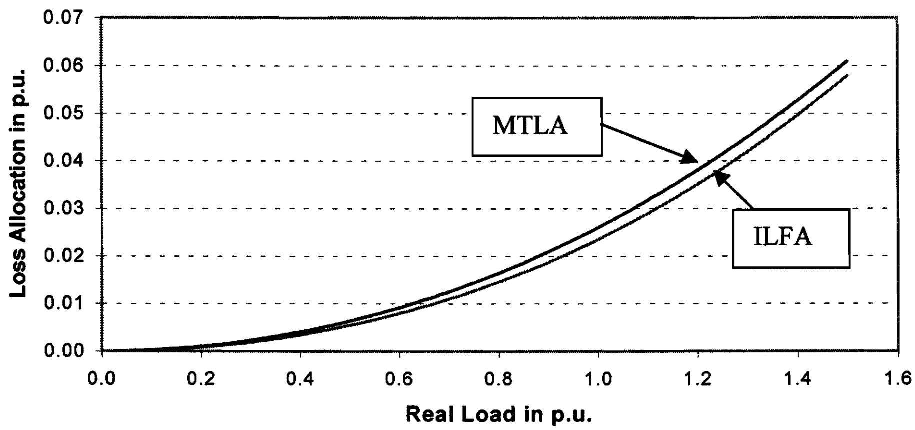

Fig. 4.2: Allocation of real transmission loss for Generator B (μ=0.6).

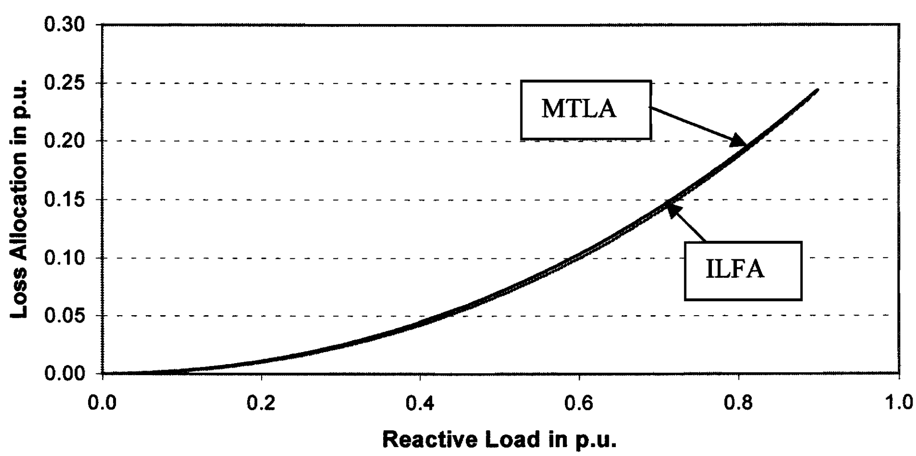

Fig. 4.3: Allocation of reactive transmission loss for Generator A (μ=0.6).

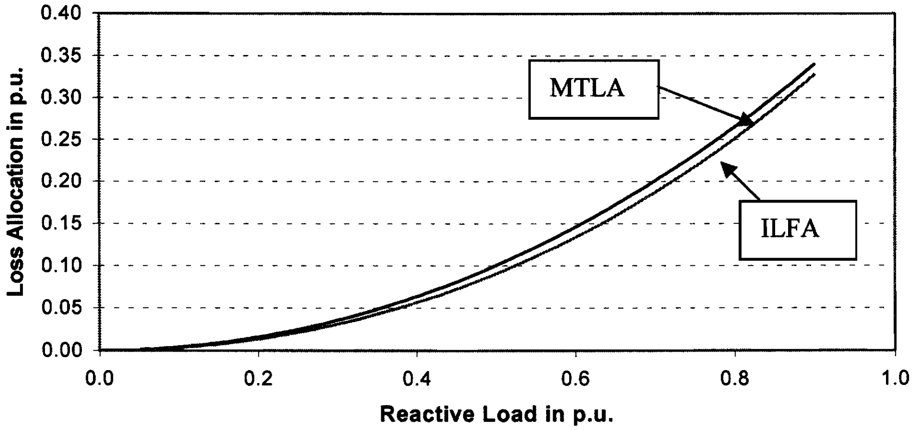

Fig. 4.4: Allocation of reactive transmission loss for Generator B (μ=0.6).

These figures represent a comparative view of transmission loss allocations obtained from the two different methods. Loss allocations for Generator A by the ILFA and the MTLA are very close for both real and reactive powers whereas those for Generator B are little different. For Generator B, losses allocated by the MTLA are slightly higher than losses allocated by the ILFA for both real and reactive powers.

4.6 Allocation of Transmission Loss by Using the Generalized MTLA

The MTLA equations were network dependent. New sets of equations are required to obtain transmission losses for every new system. On the other hand, the generalized MTLA is versatile and can be readily applied to a network of any size. A computer program has been developed to solve Equations (4.19) and (4.20) for the determination of the share of transmission losses. Different load conditions have been taken into consideration to find the transmission loss shares. The results have been compared with the shares obtained in the incremental load flow technique.

To start with the allocation process, the loads are varied from zero to their respective load demands. In the MTLA, the loads are increased step by step in a sequential manner. In any iteration, only one customer load is increased while the other loads are held fixed. Assume that load L_a is increased by a step of ΔL_a while L_b stays at its previous level. Since the load demand of Customer B (L_b) is unchanged, the resulting incremental transmission loss becomes the liability of Generator A to produce it in order to support the incremental load demand of Customer A. In the next iteration, L_b is increased by a step size of ΔL_b while L_a remains fixed. Again, generations and transmission losses are calculated and the incremental transmission loss is assigned to Generator B.

A variety of load combinations have been utilized to find the transmission loss allocation by the MTLA. Both real load demand and different reactive ratio of both customers have been varied and the transmission loss shares have been determined. The results obtained from the MTLA have been compared with those obtained from the ILFA.

4.6.1 Unequal Load and Different Reactive Ratio

In this section, the reactive ratios as well as the real loads of Customers A and B are considered different. The reactive ratio for Customer B has been varied while maintaining a constant reactive ratio for Customer A. Three different constant reactive ratios for Customer A have been considered in this evaluation. Transmission loss allocations are shown in Tables 4.3 and 4.4. The real load demands of Customer A and B are 1.5 and 0.9 p.u. respectively.

The real loss allocations for Generator A as obtained from the two methods have the maximum difference when two loads have the maximum difference in their reactive ratios. But the loss allocations for Generator B as obtained by the two methods are very close. The reactive loss allocations as obtained by ILFA and MTLA are very close too.

Table 4.3: Real Loss Allocation for unequal load and unequal reactive ratio.

Reactive Ratio		A's Share of Loss (p.u.)		B's Share of Loss (p.u.)	
μ_a	μ_b	ILFA	MTLA	ILFA	MTLA
0.6	0.3	0.0397	0.0452	0.0148	0.0150
0.6	0.4	0.0401	0.0448	0.0157	0.0162
0.6	0.5	0.0405	0.0444	0.0171	0.0177
0.5	0.3	0.0359	0.0399	0.0147	0.0151
0.5	0.4	0.0363	0.0396	0.0156	0.0163
0.4	0.3	0.0328	0.0356	0.0147	0.0152

Table 4.4: Reactive Loss Allocation for unequal load and unequal reactive ratio.

Reactive Ratio		A's Share of Loss (p.u.)		B's Share of Loss (p.u.)	
μ_a	μ_b	ILFA	MTLA	ILFA	MTLA
0.6	0.3	0.2390	0.2633	0.0836	0.0846
0.6	0.4	0.2407	0.2612	0.0899	0.0908
0.6	0.5	0.2427	0.2591	0.0974	0.0991
0.5	0.3	0.2156	0.2331	0.0832	0.0850
0.5	0.4	0.2170	0.2314	0.0894	0.0914
0.4	0.3	0.1965	0.2088	0.0825	0.0853

Figures 4.5 - 4.8 show loss allocations for different combinations of real loads and reactive ratios.

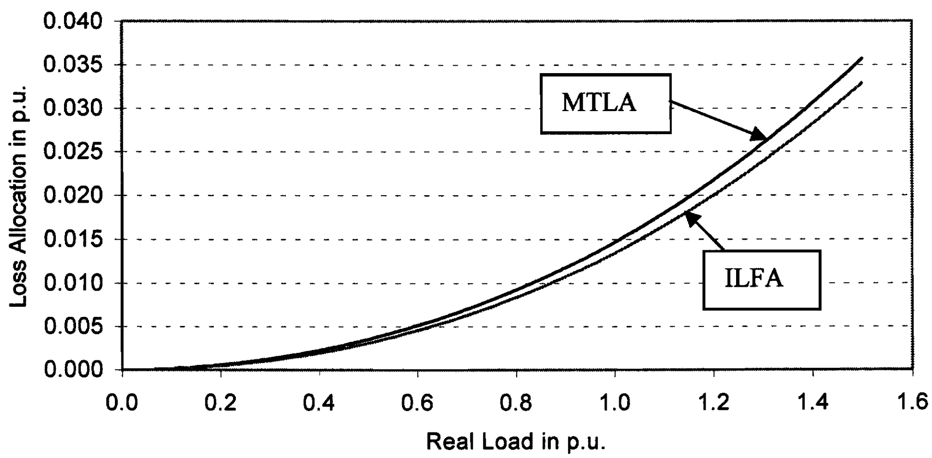

Fig. 4.5 Allocation of real transmission loss for Generator A (μ_a=0.4, μ_b=0.3).

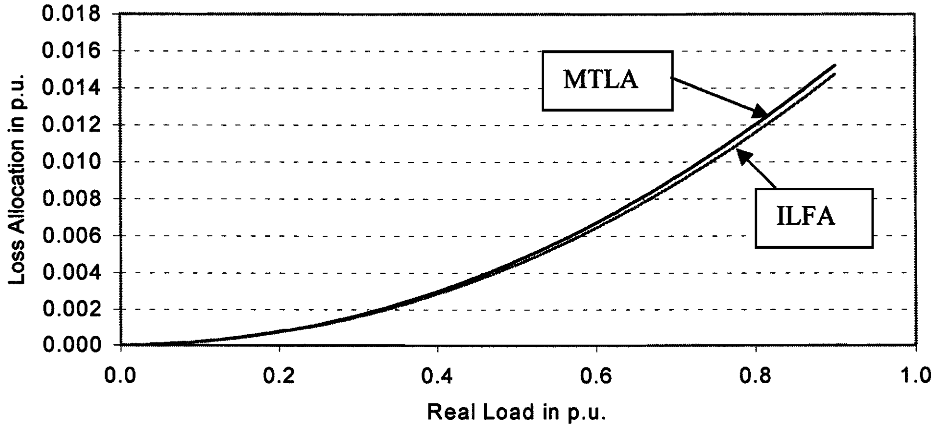

Fig. 4.6: Allocation of real transmission loss for Generator B (μ_a=0.4, μ_b=0.3).

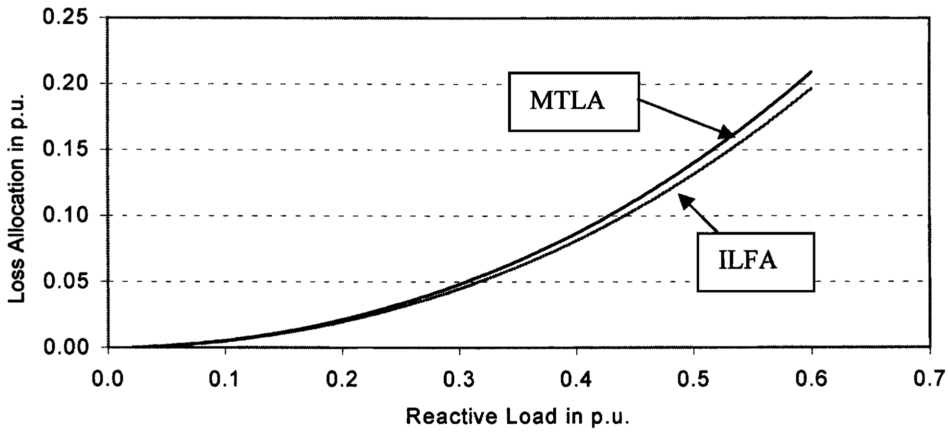

Fig. 4.7: Allocation of reactive transmission loss for Generator A (μ_a=0.4, μ_b=0.3).

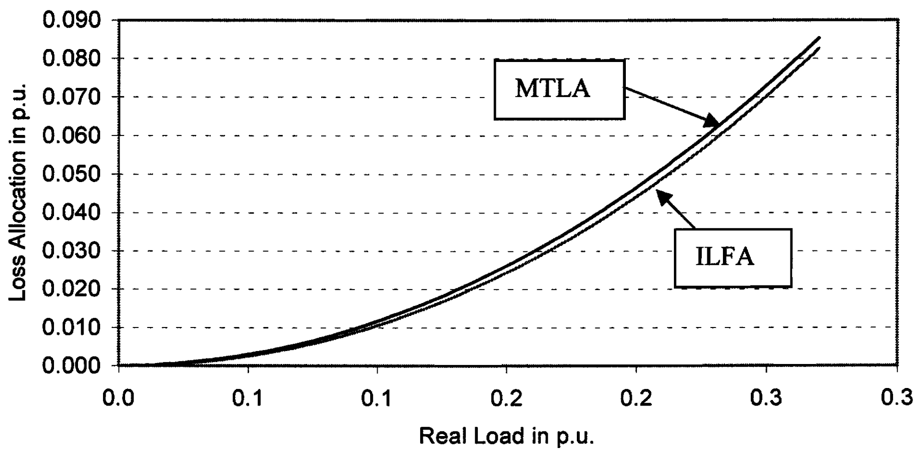

Fig. 4.8: Allocation of reactive transmission loss for Generator B (μ_a=0.4, μ_b=0.3).

From Figures 4.5 - 4.8, it can be seen that the loss allocations for Generator A as obtained by the two methods have higher differences than those of Generator B. Loss allocations for Generator A and B for two different reactive ratios have been plotted (Figures 4.17 - 4.24) and it can be noticed that Generator B's loss allocations as obtained by the two methods are very close to each other.

4.7 Comparison and Discussion

The loss allocations obtained by the two different methods, the ILFA and the MTLA, differ as the real and/or reactive load changes. It is interesting to note the effect of reactive ratio and different load demands on loss allocations. Tables 4.3 - 4.4 and Figures 4.5 - 4.8 present the loss allocations obtained by the MTLA as well as by the ILFA which clearly indicate that the differences between the loss allocations obtained by the two methods vary from nominal to significant magnitudes. The assessment of loss allocation by the MTLA is based on a number of assumptions. The assumptions have been adopted to keep the MTLA relatively simple and manageable.

In order to have a better view of the loss allocations obtained by the two methods, percentage of errors have been calculated for all load combinations. The loss allocations obtained by the ILFA have been taken as base values for error calculation.

Table 4.5 indicates the errors calculated from the two different methods used for obtaining transmission loss allocation shown in Tables 4.7 and 4.8. The real load demands of Customers A and B are 1.5 and 0.9 p.u. respectively.

Table 4.5: Error table for two different methods.

Reactive Ratio		Percentage of Error of Real Loss Allocation		Percentage of Error of Reactive Loss Allocation	
μ_a	μ_b	A	B	A	B
0.6	0.3	-13.92	-1.75	-10.15	-1.11
0.6	0.4	-11.52	-2.72	-8.48	-1.01
0.6	0.5	-9.43	-3.56	-6.75	-1.66

0.5	0.3	-11.03	-2.77	-8.11	-2.11
0.5	0.4	-8.85	-4.01	-6.62	-2.21
0.4	0.3	-8.61	-3.39	-6.23	-3.39

The maximum error occurs when the two different load demands have the maximum difference in their reactive ratios. Although the errors in loss allocations for Generator B are nominal, the errors for Generator A in some cases are high.

Tables 4.6 and 4.7 show transmission loss allocations using old network dependent equations [48] and new generalized equations. Results obtained from the generalized equations are exactly same as the results obtained using the network dependent equations.

Table 4.6: Allocation of real transmission loss using old and new sets of equations.

Load		μ		Generator	Loss Allocation	
A	B	A	B		Network Dependent Old Eqn.	Generalized Eqn.
1.5	1.5	0.6	0.6	A	0.0412	0.0412
				B	0.0609	0.0609
				Total	0.1022	0.1022
1.5	1.5	0.6	0.3	A	0.0436	0.0436
				B	0.0437	0.0437
				Total	0.0874	0.0874

Table 4.7: Allocation of reactive transmission loss using old and new sets of equations.

Load		μ		Generator	Loss Allocation	
A	B	A	B		Network Dependent Old Eqn.	Generalized Eqn.
1.5	1.5	0.6	0.6	A	0.2441	0.2441
				B	0.3398	0.3398
				Total	0.5839	0.5839
1.5	1.5	0.6	0.3	A	0.2561	0.2561
				B	0.2446	0.2446
				Total	0.5008	0.5008

4.8 Loss Allocation in the IEEE-RTS

The IEEE 24 bus Reliability Test System mentioned in Chapter 3 is utilized in this section to provide numerical examples. A mixed-mode market system has been considered where both pool and bilateral contracts exist at the same time. A hypothetical Power Pool for IEEE RTS determines the market clearing price from the supply curve of power and from the demand of total power. The bids and the market clearing price is mentioned in Chapter 3. The demand side bidding has not been considered in the IEEE system. The power demand in the system for a particular hour is considered to be 2494 MW of real power 589 MVAR of reactive power. Two bilateral contracts have been considered in the system. First contract, known as Contract A, is between the load at Bus 9 and the generators at Bus 7. The load at Bus 9 is termed as Customer A and the generators at Bus 7 are termed Generator A. At Bus 7 there are three 1.00 p.u. (real) units and these plants are already selling some energy to the pool. They have a contract with the customer at Bus 9 for supplying 1.76 p.u. of real power and 0.3621 p.u. of reactive power.

Second contract known as Contract B is between load at the Bus 19 and the generators at Bus 23. The load at Bus 19 is termed as Customer B and the generators at Bus 23 are termed Generator B. At Bus 23, there are two 1.55 p.u. (real) and one 3.50 p.u. (real) generators with a total capacity of 6.10 p.u. (real) These generators are also supplying the pool and have a bilateral contract with the customer at Bus 19 for supplying 1.81+j0.37 p.u. of apparent power.

Table 4.8 shows the loads and generations at different buses for the pool system without considering the bilateral contracts.

Table 4.8: Loads and Generations in the IEEE-RTS for the pool system (base case).

Bus	Load (p.u.)		Generation (p.u.)	
	Real	Reactive	Real	Reactive
1	1.0800	0.2200	1.5753	0.2075
2	0.9700	0.2000	1.7200	0.0511
3	1.8000	0.3700	0.0000	0.0000
4	0.7400	0.1500	0.0000	0.0000
5	0.7100	0.1400	0.0000	0.0000
6	1.3600	1.1000	0.0000	0.0000
7	1.2500	0.2500	1.0000	0.9520
8	1.7100	0.3500	0.0000	0.0000
9	0.0000	0.0000	0.0000	0.0000
10	1.9500	0.4000	0.0000	0.0000
11	0.0000	0.0000	0.0000	0.0000
12	0.0000	0.0000	0.0000	0.0000
13	2.6500	0.5400	3.9400	1.0036
14	1.9400	0.3900	0.0000	0.0000
15	3.1700	0.6400	1.5500	1.5847

16	1.0000	0.2000	1.5500	1.0991
17	0.0000	0.0000	0.0000	0.0000
18	3.3300	0.6800	4.0000	0.6108
19	0.0000	0.0000	0.0000	0.0000
20	1.2800	0.2600	0.0000	0.0000
21	0.0000	0.0000	4.0000	-0.4205
22	0.0000	0.0000	3.0000	-0.3799
23	0.0000	0.0000	3.1000	-0.1185
24	0.0000	0.0000	0.0000	0.0000

Total generations in the system are 25.4353 p.u. and 4.5899 p.u. of real and reactive power respectively. Transmission losses can be calculated from the total generations and demands in the system. Total real and reactive power losses in the network are 0.4953 p.u. and –1.3001 p.u. without considering the bilateral contracts.

In the MTLA, the loads are increased in incremental steps in a sequential manner. In any iteration, only one customer load is increased while the other loads are held fixed. Let us assume that load L_9 is increased by a step of ΔL_9 while L_{19} stays at its previous level. A computer program has been developed to find the generation and transmission loss for this condition. Since load demand of Customer B (L_{19}) is unchanged, the resulting incremental transmission loss becomes the obligation of Generator A. In the next iteration, L_{19} has been increased by a step size of ΔL_{19} while L_9 remains fixed. Again, generations and transmission losses are calculated and the incremental loss is assigned to Generator B. This is done as only L_{19} has been incremented in this iteration and Generator B is providing power to Customer B.

Generators A and B are connected to Buses 7 and 23 respectively, which have been declared as PV bus. The generations at Buses 7 and 23 have been calculated on the basis of current load (either L_9 or L_{19}) and transmission loss calculated from the previous load. Since the transmission loss for the first increment of load (L_9 or L_{19}) is unknown, the output of Generator A or B (G_7 or G_{23}) has been specified equal to ΔL_9 or ΔL_{19} during the first increment of Customer A or B's load. In the next increment of Customer A or B's load, G_7 or G_{23} has been updated as L_9 or L_{19} plus the corresponding share of the line loss from the previous iteration. This approximation works well as long as the step size remains small which is 1MW (0.01 p.u.) in this case.

Table 4.9 shows loads and required generations (loads + allocated transmission losses) at Buses 7, 9, 19 and 23 and the transmission losses. The Generators at Bus 7 are required to produce 2.7006+j1.5388 p.u. in order to supply the contracted load at Bus 9. This includes the contracted load and the transmission loss associated with supplying this load. This required generation also includes generation for the pool which is 1.0000+j1.4336 p.u. Similarly, Generators at Bus 23 are required to produce 4.9123+j1.1109 p.u. in order to fulfill the contract along with the supply to the pool.

Table 4.9: Contracted loads and generations in the IEEE-RTS determined by MTLA.

	L_9 (p.u.)	L_{19} (p.u.)	G_7 (p.u.)	G_{23} (p.u.)	Line loss (p.u.)
Real	1.76	1.81	2.7006	4.9123	0.4382
Reactive	0.3621	0.37	1.5388	1.1109	-1.5161

Table 4.10 shows the transmission loss allocation, both real and reactive, determined by the ILFA and MTLA.

Table 4.10: Loss Allocation by ILFA and MTLA.

Method	Total Loss (p.u.)		Calculated Loss Share of A (p.u.)		Calculated Loss Share of B (p.u.)	
	Real	Reactive	Real	Reactive	Real	Reactive
ILFA	0.4286	-1.5530	-0.0706	-0.2836	0.0039	0.0306
MTLA	0.4382	-1.5161	-0.0595	-0.2404	0.0024	0.0243

Table 4.10 provides a comparative presentation of loss allocations obtained by two different methods. Total real transmission loss obtained by the MTLA is 0.4382 p.u. compared to 0.4286 p.u. calculated by the ILFA. Total reactive transmission loss obtained by the MTLA is -1.5161 p.u. compared to –1.5561 p.u. determined by the ILFA. Transmission loss shares determined by the MTLA for Contract A, both real and reactive, are negative which is in agreement with the loss allocations obtained by the ILFA. This indicates that the power flows for supplying the contracted loads oppose the initial flows in some of the lines. On the other hand, transmission loss allocations for Contract B are positive as determined by both methods.

Figures 4.9-4.12 show the loss allocations obtained from two different methods. The loss allocations for both real and reactive power are plotted against their real load demand. It can be seen from Figures 4.9-4.12 that the curves of loss allocations, obtained from two methods, exhibit similar trend. The discrepancies in the graphs are discussed in Section 4.9.

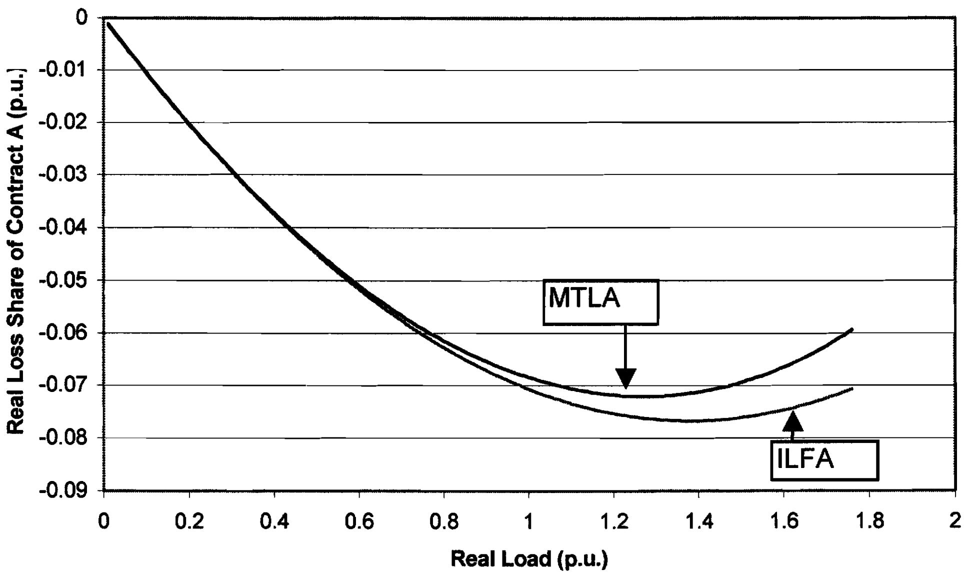

Fig. 4.9: Transmission loss allocations of real power for Contract A obtained by the ILFA and MTLA.

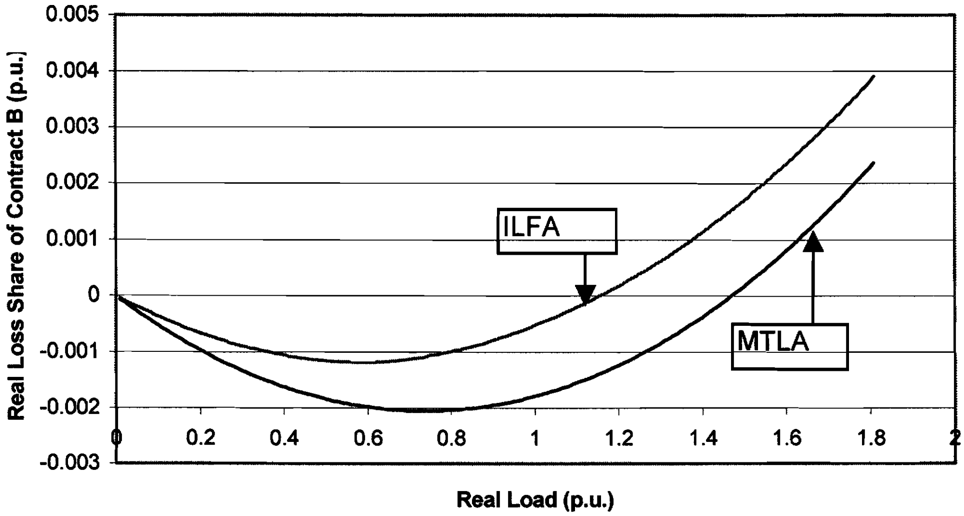

Fig. 4.10: Transmission loss allocations of real power for Contract B obtained by the ILFA and MTLA.

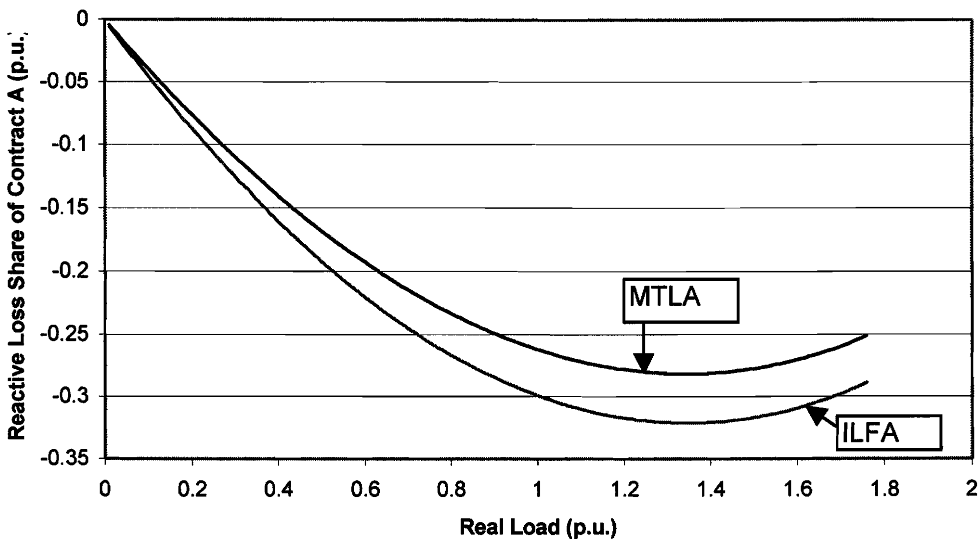

Fig. 4.11: Transmission loss allocations of reactive power for Contract A obtained by the ILFA and MTLA.

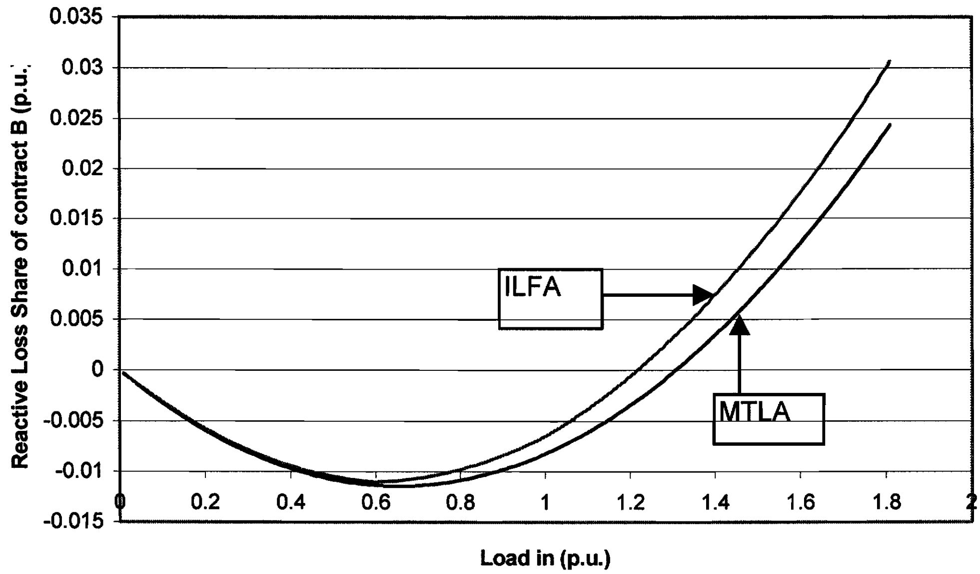

Fig. 4.12: Transmission loss allocations of reactive power for Contract B obtained by the ILFA and MTLA.

Figures 4.9-4.12 show that loss allocations obtained for the Contracts A and B using two different methods are very close. This is true for real loss allocations as well as reactive loss allocations. The difference in Loss allocations for Contract B, obtained by two methods, is higher than the difference in loss allocations for Contract A. This is because loss allocations obtained for the Contract B is very small compared to the total loss as well as Contract A's loss allocation. Contract B's loss allocations have minimal effect on the total loss allocation. In the next step loads of Contract B, both real and reactive, are increased in order to observe the change in loss allocation. The new load at Bus 19, which is in bilateral contract with Generator at Bus 23, is 2.81 p.u. real with a reactive part of 0.5744 p.u. The loads belonging to Contract A remain same. Transmission loss allocations using the ILFA and the MTLA for the increased at load at Bus 19 are shown in the Table 4.11.

Table 4.11: Loss Allocation by ILFA and MTLA with increased load at Bus 19.

Method	Total Loss (p.u.)		Calculated Loss Share of A (p.u.)		Calculated Loss Share of B (p.u.)	
	Real	Reactive	Real	Reactive	Real	Reactive
ILFA	0.4403	-1.4577	-0.0706	-0.2836	0.0156	0.1259
MTLA	0.4504	-1.4267	-0.0595	-0.2404	0.0145	0.1138

Table 4.11 shows that loss allocation for Contract B has become significantly higher because Generator at Bus 23 is supplying more power to its increased contracted load at Bus 19. It can be observed that loss allocations, both real and reactive, for Contract A remain same as in the previous case. This is obvious due to the fact that load at Bus 9 which is in contract with Generator at Bus 7 did not change. Table 4.11 also shows that transmission loss allocations obtained using two different methods are very close. Figures 4.13-4.16 show the new transmission loss allocations with increased load at Bus 19.

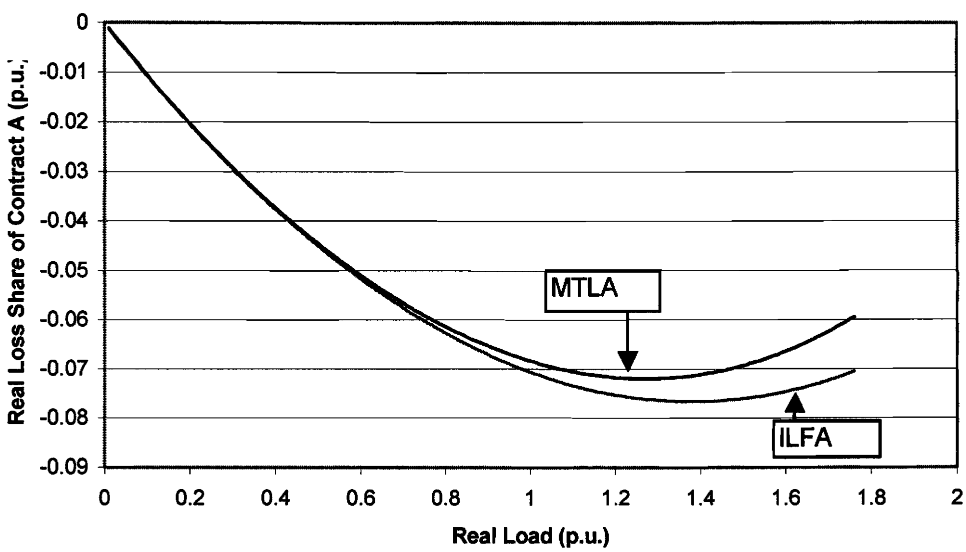

Fig. 4.13: Transmission loss allocations of real power for Contract A obtained by the ILFA and MTLA with increased load at Bus 19.

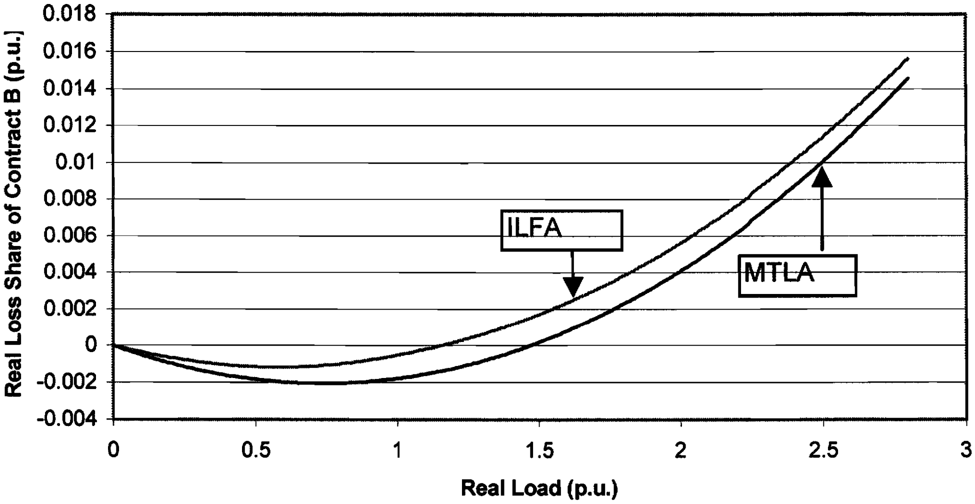

Fig. 4.14: Transmission loss allocations of real power for Contract B obtained by the ILFA and MTLA with increased load at Bus 19.

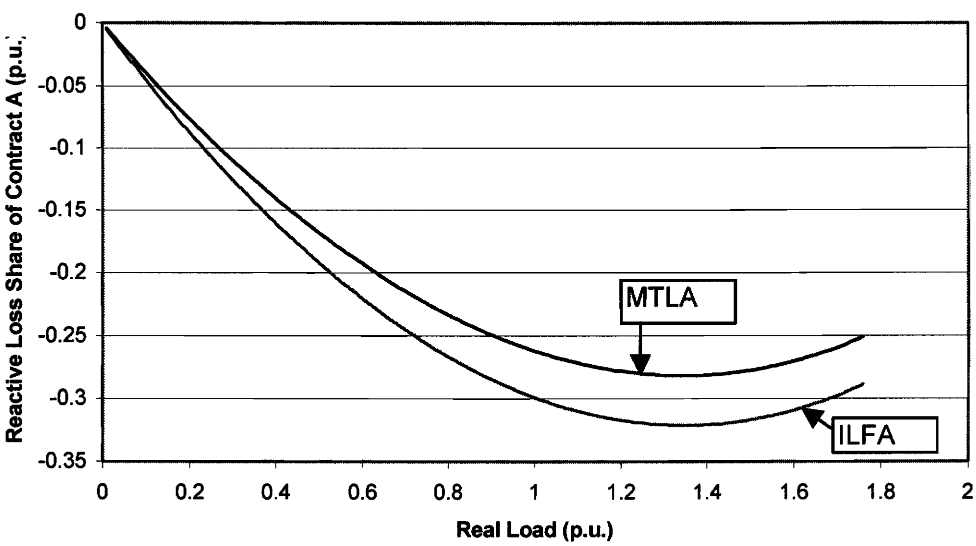

Fig. 4.15: Transmission loss allocations of reactive power for Contract A obtained by the ILFA and MTLA with increased load at Bus 19.

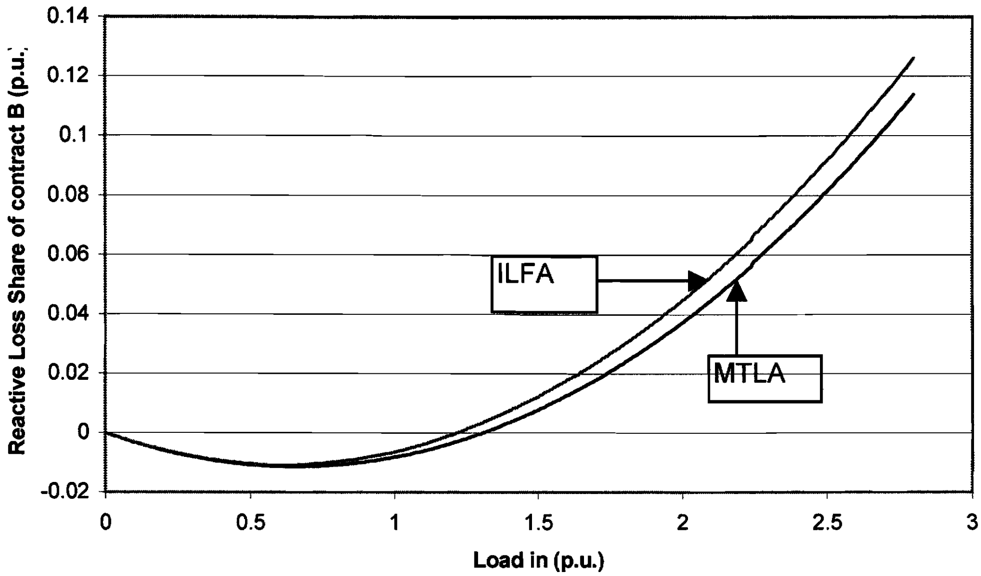

Fig. 4.16: Transmission loss allocations of reactive power for Contract B obtained by the ILFA and MTLA with increased load at Bus 19.

4.8.1 Loss Allocation involving Multiple Generator and Loads in the IEEE-RTS

The developed methods can be utilized to calculate loss allocation between two buses in a power system network. A generator may, however, be contracted to supply more than one load or a load may have contracts with multiple generators to receive power. The ILFA and MTLA can identify and calculate loss allocation for each individual transaction when a generator or a load has multiple contracts at the same time. In the IEEE system, it has been assumed that the Generator connected at Bus 7 is supplying power to loads at Buses 8, 9 and 10 and the Generator connected at Bus 23 is supplying power to Buses 10 and 19. These transactions are bilateral contracts. In order to make the calculations simple it has been assumed that all contracted loads are equal to 0.96+j 0.197 p.u. In addition to the contracted loads, Buses 8 and 10 have system loads of 1.71+j0.35 p.u. and 1.95+j0.40 p.u. respectively. Table 4.12 shows transmission loss allocation for these bilateral contracts. It should be noted that the load at Bus 10 is receiving power from two generators, one at Bus & and the other at Bus 23.

Table 4.12: Loss Allocation for multiple bilateral contracts.

Generator Bus	Load Bus	Total Bus Load (p.u.)		Loss (p.u.)	
		Real	Reactive	Real	Reactive
7	8	2.67	0.547	0.0451	0.2129
	9	0.97	0.197	-0.0206	-0.1043
	10	3.87	0.794	0.0005	0.1409
23	10	3.87	0.794	0.0442	0.5279
	19	0.96	0.197	0.0254	0.3056

4.9 Overview and Discussion

Two different methods for obtaining loss allocation obtained from, ILFA and MTLA, are utilized to obtain transmission loss allocation for two different test systems. The test system shown in Figure 3.1 is a small network and used for better understanding of the methods. In the test system, a deregulation of the network has been considered where only bilateral contract can exist. Deregulation means open competition between the generating utilities for supplying the load demands. It should work like open market for any other commodity and market should be solely controlled by demand and supply of power. The test system gives a glimpse of an extreme example of deregulation. The system consists of two bilateral contracts and a number of load combinations were used to obtain the transmission losses for these two contracts. The total load in the system is demanded by two bulk customers and supplied by two generators. The results obtained using the ILFA and the MTLA are shown in tables and graphs. It can be seen from the results that the obtained loss allocations are very close.

At present, many utilities are facing difficult challenge to transform their system from a full monopoly to a full deregulated one. Pool system is working as an in-between system. Some utilities are operating solely on pool system and some are allowing bilateral contracts along with the pool system. The IEEE-RTS shown in Appendix-A has been used in the work as a representation of the combined or mixed-mode system. The system is assumed to operate using a pool system. The suppliers compete to sell their electric power through bidding at the power pool. The market price is set by the price of the last bidder where the total generation offered equals to the total load demand. In the IEEE-RTS two bilateral contracts are considered within the existing power pool operation. The contracted generators supply their firm customers in addition they also sell power to the power pool through bidding. The resulting transmission losses for supplying the contracted power have been calculated for the contracted generators by the ILFA and the MTLA. Transmission loss allocations for the bilateral contracts in the IEEE system, obtained utilizing two different methods, follow each other closely.

The loss allocations obtained by the two different methods, the ILFA and the MTLA, for both systems, the Test system and the IEEE RTS, are shown in tabular and graphical form. Results obtained from the two proposed methods differ due to various assumptions made in the formulation of these methods. The assessment of loss allocation by the MTLA is based on a number of assumptions. These assumptions have been adopted to keep the MTLA relatively simple and manageable.

In the test system the transmission lines are considered to be short line and hence the effect of charging currents is ignored e.g. susceptance has been considered to be zero. This means reactive power support for the network is originated from the generators only. But in the IEEE-RTS, transmission lines considered to be medium lines and the charging currents are taken into consideration. This means the transmission lines provide considerable amount of reactive power support for the system network. The line susceptances are considered to be lumped at the end of the lines. In MTLA, it is required to find the initial conditions, real and reactive power generations, voltages and bus angles in the network, for the calculation of loss allocations. It has been assumed that reactive power support from the transmission lines are lumped at the generating buses only to make the method simple. This assumption, however contributes some differences in the loss allocation obtained the ILFA and MTLA.

The mathematical method described in this chapter has some unique characteristics. One of the important characteristics of the MTLA is it can be used for contracts between any buses in a power system network. If a load at a certain bus contracts several generators at different buses for supplying its demand, the MTLA can determine loss allocations separately for each of the generators. The Figure 4.17 shows generation and load matrix in a power system network. In this figure rows corresponds to generating buses while loads are places in the columns in accordance with the MTLA. Any matrix element (i.e. 2,n) means the load at bus n is receiving partial or full support from the generator connected at bus 2. The load at bus n may have other supplier and in that case the column n will have non-zero value in other rows. If column has non-zero value in the ith row then it indicates that the load at bus n also receiving power supply from the generator connected to bus i.

	LOAD								
GENERATION	1,1	1,2	1,3	…	…	1,i	1,j	.	1,n
	2,1	2,2	2,3	…	…	2,i	2,j	.	2,n
	3,1	3,2	3,3	…	…	3,i	3,j	.	3,n
	…	…	…	…	…	…	…	…	…
	…	…	…	…	…	…	…	…	…
	i,1	i,2	i,3	.	.	i,i	i,j	.	i,n
	j,1	j,2	j,3	.	.	j,i	j,j	.	j,n
	…	…	…	…	…	…	…	…	…
	n,1	N,2	n,3	.	.	n,i	n,j	.	n,n

Fig. 4.17: Generation-load matrix.

The developed method, MTLA, can be used to find the loss allocation for each of the transaction individually. For example, load at bus 3 has bilateral contracts with generators at buses 1,2 and j. Now using Equation (4.27) transmission loss allocations, both real reactive, can be obtained for bilateral contracts between buses 3 and 1, buses 3 and 2 and buses 3 and j. Similarly a generator can supply multiple loads at the same time according to bilateral contracts and loss allocations for each contracts can be calculated separately. For example, generator at bus i is supplying power to loads connected to 2, 3 and n under bilateral contracts. The MTLA allows the generator at bus i to know how much transmission losses occur, both real and reactive, for supplying loads at buses 2, 3 and n individually. Thus the implementation of the MTLA gives a chance to know all individual loss allocations due to individual bilateral contracts in a power system network right away.

CHAPTER 5: TRANSMISSION LOSS IN THE IEEE-RTS - UNDER DIFFERENT SCHEMES

5.1 Power System Operation

A Power system is a composite entity. Its three functional areas are generation, transmission and distribution. The combination of these three utilities and their optimized utilization is the goal of power system operation. This goal is achieved through several operating procedures such as load forecast, unit commitment and finally load dispatch.

Loads in a network follow some dynamic patterns and go high and low at different times of a day. Load forecast predicts the nature of load demand from load patterns and coming events with a high accuracy. The utilities use this prediction for determining the number of generating units required to meet the anticipated load, an essential activity of a power system, generally known as unit commitment.

Unit commitment dictates the number of generating units to be in the spinning mode to meet load demand during the 24 hours. It also states the order of the units to be engaged in power production according to the production cost of these units and starting time of the units. Production cost varies from unit to unit depending on their design, age and working principle. The production costs of hydro units are far less than those of the gas turbine and thermal units. It costs much more to produce energy in a gas turbine unit compared to that of a similar size thermal unit. The starting time of gas turbine and hydro units are much shorter than conventional thermal plants. These two factors, production cost and starting time, determine the allocation of load among various units. This activity is commonly known as load dispatch.

Once the load dispatching schedule has been prepared, the need for an AC load flow analysis comes into the picture. An AC load flow analysis provides bus voltages, line flows, power mismatch etc. and helps to determine the feasibility of a load dispatching schedule.

5.1.1 Economic Load Dispatch

The main objective of an economic load dispatch is to distribute the system's total load plus transmission loss to its committed generating units in such a way that the total production cost becomes minimum. Since the load in a system varies on a continuous basis, the activity of economic load dispatch therefore distributes the load and transmission loss on a continuous basis. It is a cost minimizing activity with several constraints. These constraints originate from the requirements and limitations imposed either by network design and or operating conditions. Some typical constraints are reactive power flow, voltage profile and line capacity.

At present, power generating plants are mostly run by fossil fuel and water force. In fossil fuel-run power stations, fuel cost is the major source of expenditure for the generation of real power. But for hydro stations the fuel cost is apparently zero. Operating characteristics of a typical thermal power station is usually described by a quadratic cost curve in Fig. 5.1. The parameters of the quadratic cost function can be obtained from experimental data [72,75,76,77].

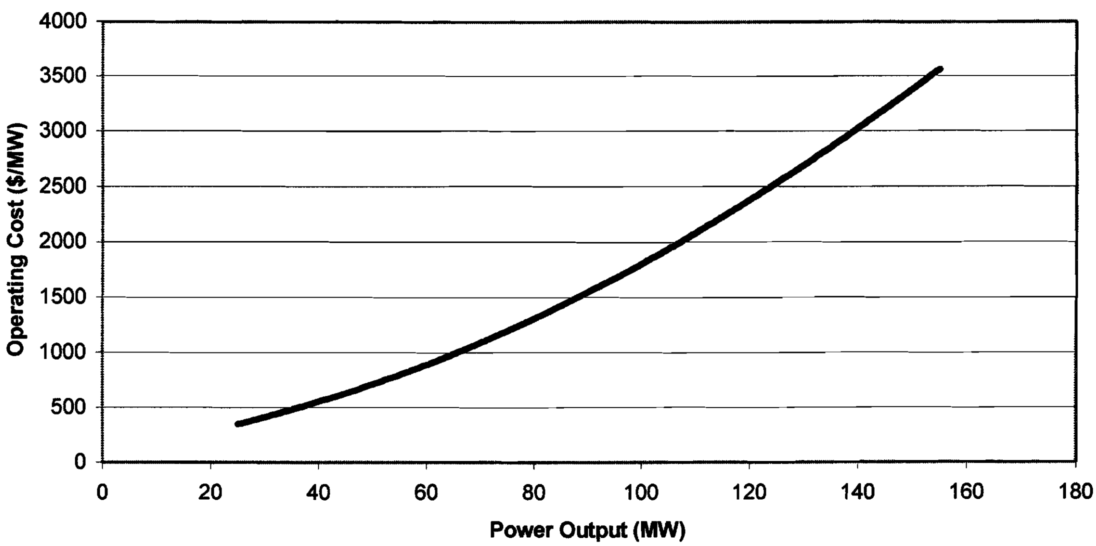

Fig.5.1: Typical cost curve of a thermal power station.

$$F = aP^2 + bP + c \qquad \$/\text{Hr} \tag{5.1}$$

where,

P = power generation

a, b and c = cost parameters

The output of each generator is constrained by its minimum and maximum output limits. The minimum limit is determined either from economical or technical infeasibility considerations.

Marginal cost of power production can easily be obtained as:

$$\frac{dF}{dP} = \lambda = aP + b \qquad \$/\text{MwHr} \tag{5.2}$$

Marginal cost is also known as unit (plant) Lambda (λ). Marginal cost represents cost of one additional unit of power. The goal is to operate all units with the same value of Lambda in order to make the system operation the most economic. The optimal point of operation of a network will be achieved when all generators will have same marginal cost. If Transmission loss P_l is also considered then the marginal cost becomes:

$$\lambda = \frac{dF}{dP} + \lambda \frac{\partial P_l}{\partial P} \tag{5.3}$$

It has been considered that power loss in transmission network is happening at the same rate of Lambda. For economic dispatch the system wide marginal cost (λ_s) becomes:

$$\lambda_s = \frac{dF}{dP}\left(\frac{1}{(1 - \frac{\partial P_l}{\partial P}}\right) \tag{5.4}$$

Economic load dispatch for a power system network can be obtained using different existing methods such as Shoult's technique, First Order Gradient Method and Dynamic Programming Method. In this thesis Shoult's method has been used to obtain the economic load dispatch [74]. This method is very popular and widely used.

5.2 Transmission Loss Allocation in a Bilateral Contract

Transmission loss in an electric power system network plays an important role in the operation of the network. In an old-fashioned monopoly system, the operation of the network emphasizes optimal operation or minimal cost, not on minimal transmission loss. In a monopoly system, transmission loss is the liability of the whole utility and the operator distributes the loss among its generators according to the output dictated by the economic load dispatch.

Under deregulation, a generating plant can sign a bilateral contract with any customer in the system. According to a bilateral contract, the generating plant has the responsibility of providing power to the contracted load. Any flow of electricity incurs some losses and the contracted generating plant needs to compensate for the loss too. Transmission loss allocations, both real and reactive, can be determined by the methods described in Chapter 3 and 4. Once the generator knows its share of transmission loss for supplying a particular load it can consider different options for compensating for that loss. The possible different choices for a generator are:

- Demand and Loss Provided by the Same Generator.
- Loss Provided by Different Generator or ISO.
- Contracted Generator as a Part of Economic Load Dispatch.

The following sections describe different options of a generating plant for compensating the transmission loss. IEEE 24–bus Reliability Test System has been used to study different scenarios. The algorithm for economic load dispatch incorporating the bilateral contracts is summarized below:

1. Obtain the basic load flow for the system without considering the bilateral contracts.
2. Use the data obtained in basic load flow in the Economic Load Dispatch program for the calculation of optimal power generation.
3. Use either ILFA or MTLA to calculate transmission loss allocation for the bilateral contracts.

4. Use loss allocation results for new load flow considering the bilateral contracts.

5. Use data obtained in step 4 to recalculate the optimal generation by Economic Load Dispatch program for different schemes.

5.2.1 Demand and Loss Provided by the Same Generator

A generator in a bilateral contract has the responsibility to supply the contracted load as well as associated transmission losses. The contracted generator is allowed to make as many contracts as it wants and it is also able to know transmission losses related to each transaction of power. After knowing the transmission losses related to each transaction, the generator can decide whether to provide the losses from its own sources or buy it from different sources. This decision depends on various issues such as relative distance between the plant and the contracted loads, demand and market price of power etc.

The network operation in this example is considered to be done by Economic Load Dispatch. A basic load demand of 2400 MW is considered in the IEEE-RTS network before the bilateral contract comes in. Load demands, real and reactive, and priority loading order of generating units [75,76] are shown in Table 5.1 and 5.2 respectively.

Table 5.1 Loads connected to different buses in the IEEE-RTS.

Bus	Load	
	Real (MW)	Reactive (MVAR)
1	108	22
2	97	20
3	180	37
4	74	15
5	71	14
6	136	28
7	125	25
8	171	35
9	000	00
10	195	40
11	000	00

12	000	00
13	265	54
14	100	39
15	317	64
16	100	20
17	000	00
18	333	68
19	000	00
20	128	26
21	000	00
22	000	00
23	000	00
24	000	00

Table 5.2: Priority loading order of generating units.

Loading Order	Unit Identification	Capacity (MW)	Bus
1	#1	50	22
2	#2	50	22
3	#3	50	22
4	#4	50	22
5	#5	50	22
6	#6	50	22
7	#7	400	18
8	#8	400	21
9	#9	350	23
10	#10	197	13
11	#11	197	13
12	#12	197	13
13	#13	155	16
14	#14	155	23

15	#15	155	23
16	#16	155	15
17	#17	100	7
18	#18	100	7
19	#19	100	7
20	#20	76	1
21	#21	76	1
22	#22	76	2
23	#23	76	2
24	#24	12	15
25	#25	12	15
26	#26	12	15
27	#27	12	15
28	#28	12	15
29	#29	20	1
30	#30	20	1
31	#31	20	2
32	#32	20	2

Shoult's method has been used to obtain the Economic Load Dispatch for this system. The total transmission loss, system Lambda (λ_s) and running cost for supplying the load demand of 2400 MW are 62.1753 MW, 12.55664 \$/MWHr and 26960.89 \$/Hr. Marginal cost of real power for the system is 12.55664 \$/MWHr.

A bilateral contract can be considered once the data for the pool operation are obtained. Consider a bilateral contract between the generators connected at Bus 7 and the customer connected at Bus 9. The Customer has a load demand of 175 MW of real power and 36 MVAR of reactive power. According to the bilateral contract generators at Bus 7 are responsible for supplying the load at Bus 9. Generators at Bus 7 are considering supplying both load and its associated transmission losses. Transmission losses related to the bilateral transaction, real and reactive, are determined using one of the developed methods. Generators at Bus 7 have a negative transmission loss allocation if they supply the load at Bus 9 which is – 4.06866 MW. The combined output of the generators at Bus7 is:

$$G_7 = 120 + 175 + \text{loss} \quad \text{MW}$$

The Generators at Bus 7 are supplying 120 MW to the pool and 175 MW to Bus 9 and the associated loss. Transmission loss is negative in this case. Economic Load Dispatch program has been run again with a constraint that generators at Bus 7 are providing power to the contracted load at Bus 9. The total transmission loss, system Lambda (λ_s) and running cost for supplying the load demand of 2575 MW are 56.3163 MW, 12.32662 \$/MWHr and 30823.06 \$/Hr. Marginal cost of real power for the system is 12.32662 \$/MWHr.

Transmission loss has been found to be negative for the bilateral contract between generators at Bus 7 and load at Bus 9. The negative transmission loss is the result of counter-flow and the bilateral contract should not have the full benefit of transmission loss reduction. It should be divided equally between the pool and the bilateral contract. The total transmission loss, system Lambda (λ_s) and running cost for supplying the load demand are 56.3915 MW, 12.31884 \$/MWHr and 30846.22 \$/Hr. Marginal cost of real power for the system is 12.31884 \$/MWHr when the bilateral contract is awarded half of the transmission loss reduction (2.03433 MW).

5.2.2 Loss Provided by a Different Generator or ISO

A customer may want to buy power from a remote generator and this decision may be taken because of various considerations e.g. encouraging green power, subsidizing remote generating units etc. But the contracted generating unit may want to supply the contracted load only. In this situation the generating plant will depend on some other generating plants which are situated near its contracted customer. The contracted plant might want a contract with a local plant for supplying the loss occurred due to its contract with the customer. This would be a trilateral contract.

In another model, the generating utilities and the customers can sign bilateral contracts and the ISO will be responsible for supplying the loss occurred due to all contracts. The ISO will determine a suitable plant based on location and price of electricity for buying energy to compensate for the loss incurred by the bilateral contracts. All bilateral parties involved would pay the ISO for their respective allocated losses.

In this situation the contracted generators at Bus 9 will produce

$G_7 = 120 + 175 \quad \text{MW}$

The total transmission loss, system Lambda (λ_s) and running cost for supplying the load demand are 56.4878 MW, 12.30359 \$/MWHr and 30866.68 \$/Hr. Marginal cost of real power for the system is 12.30359 \$/MWHr.

5.2.3 Contracted Generator as a Part of Economic Load Dispatch

A generator, in bilateral contract with a customer, may want to include itself in the economic load dispatch schedule of the power system network that it uses to supply its contracted load. When a generator becomes a part of the economic load dispatch schedule of a network, the term bilateral contract apparently does not exist anymore. But from the customer's point of view, there is nothing to object as long as his load demand is satisfied. From the Generator's point of view, it would want to be a part of the whole network, if it proves to be profitable. The generator would pay the utility/ISO for the increase in total operating cost due to its inclusion in the system. The total transmission loss, system Lambda (λ_s) and running cost for supplying the load demand of 2575 MW are 57.8316 MW, 13.59401 \$/MWHr and 29509.67 \$/Hr. Marginal cost of real power for the system is 13.59401 \$/MWHr when the contracted generator includes itself in the Economic Load Dispatch schedule.

5.2.4 Comparison of Costs With Different Schemes

Different schemes of transmission loss compensation have been considered where a generator is in a bilateral contract for supplying a contracted load. Generators connected to Bus 7 are supplying the demand of customer connected at Bus 9. Table 5.3 shows a comparative study of cost for supplying 175 MW of power demand by the customer at Bus 9.

Table 5.3: Comparative Cost Study for Different Schemes.

Scheme		Total Cost ($/Hr)	Cost of Contract ($/Hr)
Base Case		26960.89	–
Loss Supplied by Generator	Full Loss	30823.06	3862.17
	Half Loss (Counter-Flow only)	30846.22	3885.33
Loss Supplied by the ISO		30866.68	3905.79
Contracted Generator as a Part of ELD		29509.67	2548.78

From Table 5.3 it is evident that the lowest cost of bilateral contract can be achieved if the contracted generator becomes part of the Economic Load Dispatch scheme. The lowest cost in this case is 2548.78 $/hr

Another load condition has been considered where positive transmission loss allocation is involved. The pool has a base load of 2820 MW of real power and 567 MVAR of reactive power. A bilateral contract is assumed between generators at Bus 7 and load at Bus 9. The customer at Bus 9 has a load demand of 175 MW of real power and 36 MVAR of reactive power. Total cost from different schemes are shown in Table 5.4.

Table 5.4: Comparative Cost Study for positive transmission loss allocation.

Scheme	Total Cost ($/Hr)	Cost of Contract ($/Hr)
Base Case	33389.8	–
Loss Supplied by Generator	36998.5	3608.7
Loss Supplied by the ISO	36993	3603.2
Contracted Generator as a Part of ELD	36929.3	3539.5

In case of positive transmission loss allocation, the lowest cost of providing power to a contracted load is originated from the scheme where the contracted generator is a part of Economic Load Dispatch. In this case, the lowest cost is 3539.5 $/Hr.

It has been found that if the contracted generator includes itself in the Economic Load Dispatch schedule then it will have the minimum cost. In this scheme, the contracted generator will have no control over supplying power to its contracted load rather it will depend on the decision of the ISO. This may impact the reliability of supply to the contracted load.

5.3 Counter-flow

In an electrical power system network, it is possible that one or more generators may set a flow of power that opposes the flow of the network and thereby decreases the overall transmission loss. This flow which opposes the initial flow in a particular transmission line is sometimes termed as counter-flow. Logically, a generator can contribute to a decrease in transmission loss only when there exists an initial flow in the opposite direction by another generator. Without the initial generator there would be no so called Counter-flow in the system. In case of bringing in new generation in the system, the new generator as well as the old ones would therefore be responsible for such overall reduction of transmission loss.

The allocation of transmission loss can be attributed to the generators positively of negatively based on the sequences in which the generators are brought in the system. The same generator can be attributed different loss allocation based on various sequences. The concept of counter-flow would be flawed if one does not consider the relative position of generators and loads and the sequences they are brought in the system. It is obvious that considering the numerous possible sequences in a power system network, the loss allocation would not have any unique solution unless the participating utilities reach an agreement.

5.3.1 Counter-flow in a Simple System

Let us consider a simple hypothetical power system network shown in Fig.5.2. The system has two Generators Gen_a and Gen_b connected to Buses 1 and 2 respectively and two Customers A and B connected to Buses 2 and 1 respectively. A transmission line connects Buses 1 and 2. Customer A has a load demand of 1+j0.5 p.u. and Customer B has a load demand of 1.6+j0.8 p.u.

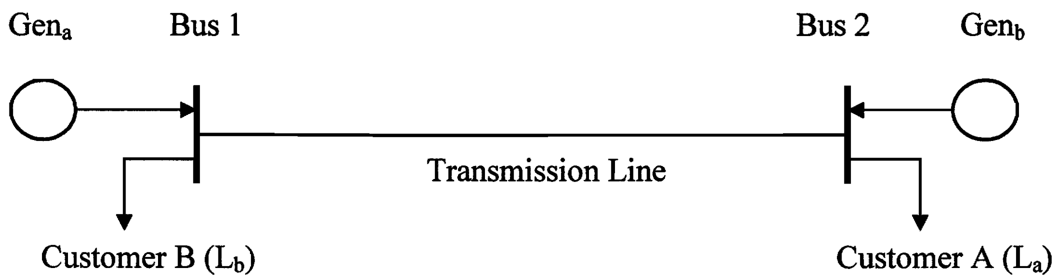

Fig. 5.2: A simple power system network.

Let us consider a bilateral contract between Gen_a and Customer A. According to the contract Gen_a will supply the load demand of Customer A and the related transmission loss. A similar bilateral contract exists between Gen_b and Customer B. The total transmission loss (Pl) depends on the net flow in the transmission line.

The system shown in Fig.5.2 is pretty simple and has counter-flow. If parties related to the second contract (between Gen_b and Customer B) are omitted from the system then there will be only one generator (Gen_a) and one customer (Customer A). The system is shown in Fig.5.3.

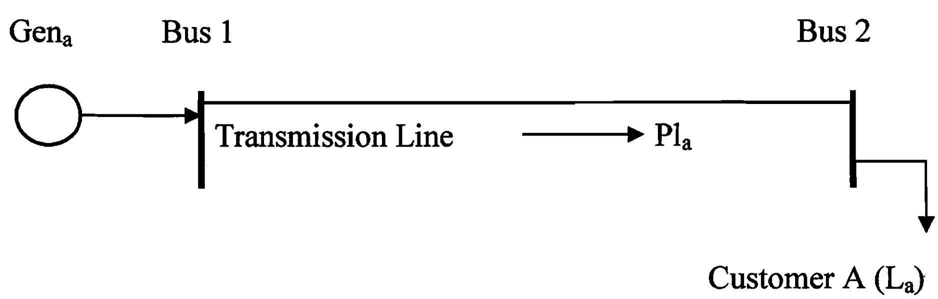

Fig. 5.3: A simple power system network with Customer A only.

It is obvious from Fig.5.3 that transmission loss (Pl_a) related to the power demand of Customer A is provided by Gen_a according to the bilateral contract.

Similarly Gen_a and Customer A (Contract A) can be omitted and the system will be left with Gen_b and Customer B (Contract B) which is shown in Fig.5.4. In this case, transmission loss (Pl_b) is associated with the supply of load demand of Customer B.

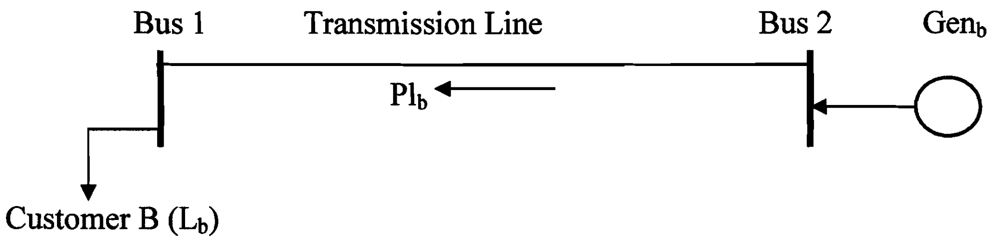

Fig. 5.4: A simple system network with Customer B only.

It is obvious that Pl will be less than Pl_a or Pl_b. Now if we bring in the contract B (Gen_b and Customer B) into the system shown in Fig.5.3 then Pl will be equal to the difference between Pl_a and Pl_b. The power will flow in the direction of bigger load. This would happen because of the counter-flow by the second contract brought into the system.

A system with only one generator and one customer cannot claim or contribute to the transmission loss reduction, as there will be no transmission loss reduction. To have counter-flow in a system there has to have opposing flows in one line which will contribute to the reduction of transmission loss. In the system shown in Fig.1 both contracts are contributing to the counter-flow and hence no single generator can claim the whole benefit of it. We have seen that the controversy about who will get the benefit of counter-flow depends on the sequence of the contracts entering into the system. It is quite logical to distribute the benefit of counter-flow among the parties involved in the counter-flow.

The concept of counter-flow is embedded in the methods to allocate transmission losses discussed in Chapters 3 and 4. For this 2-bus system, the demand of Customer A was incremented first and then that of Customer B. Results were also obtained by reversing the loading sequence and are presented in Table 5.5.

Table 5.5: Transmission loss allocations with two different transaction sequences.

Sequence	Real L_a (p.u.)	Real L_b (p.u.)	Total Loss (p.u.)	Calculated Loss Share of A (p.u.)	Calculated Loss Share of B (p.u.)
A-B	1.0	1.6	0.0118	0.0154	-0.0035
B-A	1.0	1.6	0.0117	0.0106	0.0012

From Table 5.5, it can be seen that loss share of both contracts changed when the loading sequence was reversed. However, none of the generators should claim the full benefit of the transmission loss reduction.

5.3.2 Allocation of Benefit of Transmission Loss Reduction

Existence of counter-flow depends on the relative positions of load and generators in a network. It is important to find first whether counter-flow exists in a system or not. The test system shown in Chapter 3 is used here for the study of counter-flow. It has been assumed that bilateral contracts exist between Generator A and Customer A and between Generator B and Customer B. As a result of the contracts, Generator A will supply the load demand of Customer A, and Generator B will supply the demand of Customer B. The generators supply their contracted loads alongwith the associated transmission losses. Customers A and B's real load demand are 1.5 p.u. and 0.9 p.u. respectively. Both loads have a reactive ratio of 0.6.

The methods reported in this thesis can take counter-flow into account during the assessment of transmission loss allocations. During the assessment process, customers' loads are incremented in a sequential way. In this study, first, Customer A's load is incremented and then Customer B's load is incremented. This is termed as A-B sequence and corresponding transmission losses have been calculated. This sequence has been changed to B-A and transmission losses have been obtained with this changed sequence. Results are shown in Tables 5.6 and 5.7.

Table 5.6: Transmission loss allocations with two different transaction sequences.

Sequence	Real L_a (p.u.)	Real L_b (p.u.)	R.ratio μ	Calculated Loss Share of A (p.u.)		Calculated Loss Share of B (p.u.)	
				Real	Reactive	Real	Reactive
A-B	1.5	0.9	0.6	0.04055	0.24304	0.05795	0.32778
B-A	1.5	0.9	0.6	0.04044	0.24285	0.05807	0.32801

Table 5.7: Calculated generations with two different transaction sequences.

Sequence	Real L_a (p.u.)	Real L_b (p.u.)	R.ratio μ	Generation of A (p.u.)		Generation of B (p.u.)	
				Real	Reactive	Real	Reactive
A-B	1.5	0.9	0.6	1.54144	1.46358	1.55707	0.90723
B-A	1.5	0.9	0.6	1.53984	1.46388	1.55867	0.90698

It is clear from the results shown in Tables 5.6 and 5.7 that the effect of transaction sequence is negligible as long as the incremental step is kept small. Transmission loss allocation obtained from two different loading sequences are virtually same due to the fact that there is no counter-flow in this system. This happened because of the relative position of Generator A and B and their respective loads.

In order to show the effect of counter-flow the test system has been modified. An additional generator (C) has been connected at Bus 3. Since Bus 3 has become a voltage controlled Bus, Bus 5 has been removed. Bus 6 has been renamed as Bus 5 and an additional load (Customer C) is connected at this bus. The modified system is shown in Fig.5.5. Generator C is in bilateral contract with Customer C for supplying its demand. Customers A, B and C's real load demands are 0.75 p.u., 0.75 p.u. and 1.5 p.u. respectively. To keep the calculation simpler we assume that the reactive ratio for all three loads to be 0.6. Number of loading sequences will change according to the number of contracts in a network. With the addition of a new bilateral contract there will be six different combinations of loading sequences e.g. A-B-C, A-C-B etc. Table 5.8 shows the loss allocations based on all six different sequences.

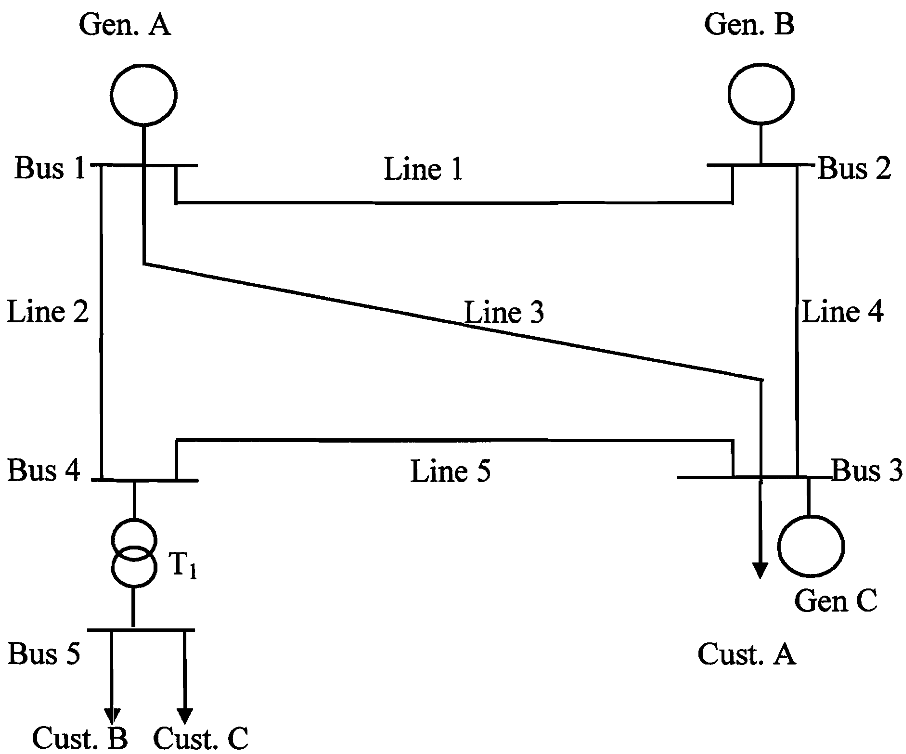

Fig. 5.5: Test system network with additional generator (C) and load (C).

Table 5.8: Transmission loss allocations with varying incremental sequence of loads.

Sequence	Total Real Loss (p.u.)	Calculated Loss Share of A (p.u.)		Calculated Loss Share of B (p.u.)		Calculated Loss Share of C (p.u.)	
		Individual	Avg.	Individual	Avg.	Individual	Avg.
A-B-C	0.0530	0.0174	0.0164	0.0206	0.0212	0.0149	0.0153
A-C-B	0.0530	0.0174		0.0209		0.0147	
B-A-C	0.0530	0.0176		0.0207		0.0147	
B-C-A	0.0529	0.0142		0.0207		0.0180	
C-A-B	0.0530	0.0176		0.0206		0.0148	
C-B-A	0.0530	0.0142		0.0239		0.0148	

It can be seen from Table 5.8 that in most of the sequences transmission loss allocations remain same. Only in two sequences, B-C-A and C-B-A, transmission loss allocations have been changed and thus confirm the presence of counter-flow in this system. Counter-flow cannot exist with a single source in a network. It requires at least two sources in a network which might have counter-flow. Hence, it is obvious to divide the benefit of counter-flow among the contributing generators. Table 5.8 shows the average of the transmission loss allocation obtained from six different loading schedules.

In the next phase, another load (Customer D) has been brought into the system at Bus 5. Customer D is assumed to have a bilateral contract with Generator C. Customer C is connected at Bus 3. The real load demand of Customers A, B C, and D are 0.75 p.u., 1.0 p.u., 0.75 p.u. and 0.5 p.u. respectively. The reactive ratio of all four loads is assumed to be 0.6. The system is shown in Fig. 5.6. Since there are four customers in the system, the total combination of sequences in which loads can be incremented is 24. Table 5.9 shows the result of loss allocations obtained from these 24 combinations.

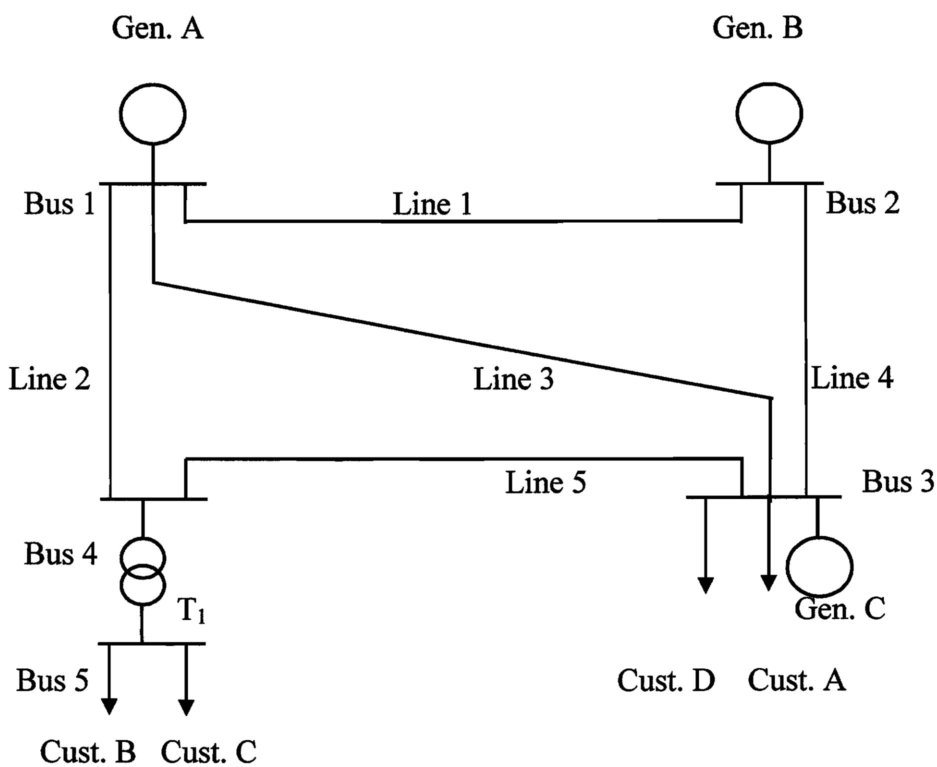

Fig. 5.6 : Test system network with additional load (D).

Table 5.9: Transmission loss allocations with varying incremental sequence of loads.

Sequence	Total Real Loss (p.u.)	Calculated Loss Share of A (p.u.)		Calculated Loss Share of B (p.u.)		Calculated Loss Share of C (p.u.)	
		Real	Reactive	Real	Reactive	Real	Reactive
A-B-C-D	0.0504	0.0027	0.0009	0.0390	0.2354	-0.0087	0.0516
A-B-D-C	0.0504	0.0115	-0.0012	0.0389	0.2353	-0.0001	0.0539
A-C-B-D	0.0504	0.0028	0.0016	0.0442	0.2347	0.0034	0.0520
A-C-D-B	0.0504	0.0028	0.0015	0.0411	0.2346	0.0065	0.0520
A-D-B-C	0.0504	0.0090	-0.0007	0.0415	0.2368	0.0000	0.0520
A-D-C-B	0.0505	0.0027	0.011	0.0478	0.2354	0.0000	0.0520
B-A-C-D	0.0504	0.0030	0.0006	0.0410	0.2338	0.0064	0.0537
B-A-D-C	0.0505	0.0031	0.0007	0.0480	0.2344	-0.0005	0.0534
B-C-A-D	0.0503	0.0091	-0.0010	0.0410	0.2336	0.0001	0.0556
B-C-D-A	0.0504	0.0026	0.0002	0.0391	0.2359	0.0088	0.0519
B-D-A-C	0.0505	0.0026	0.0004	0.0502	0.2347	-0.0022	0.0536
B-D-C-A	0.0504	0.0113	-0.0018	0.0391	0.2361	0.0000	0.0538
C-A-B-D	0.0504	0.0033	0.0002	0.0396	0.2270	0.0075	0.0608
C-A-D-B	0.0504	0.0093	0.0012	0.0411	0.2347	0.0000	0.0529
C-B-A-D	0.0505	0.0031	0.0009	0.0474	0.2344	0.0000	0.0532
C-B-D-A	0.0505	0.0026	0.0006	0.0500	0.2342	-0.0021	0.0539
C-D-A-B	0.0504	0.0029	0.0002	0.0396	0.2368	0.0079	0.0510
C-D-B-A	0.0504	0.0025	0.0001	0.0412	0.2351	0.0067	0.0529
D-A-B-C	0.0504	0.0030	0.0004	0.0396	0.2367	0.0078	0.0509

D-A-C-B	0.0505	0.0030	0.0006	0.0500	0.2353	-0.0025	0.0529
D-B-A-C	0.0504	0.0026	0.0002	0.0423	0.2371	0.0055	0.0509
D-B-C-A	0.0504	0.0089	-0.0015	0.0415	0.2371	0.0000	0.0526
D-C-A-B	0.0505	0.0025	0.0002	0.0479	0.2358	0.0000	0.0525
D-C-B-A	0.0504	0.0115	-0.0018	0.0388	0.2368	0.0001	0.0530

Table 5.9 shows that transmission loss allocations changed in some occasions confirming the presence of counter-flow in the system.

As mentioned earlier, counter-flows exist in a system when the power flows of generators oppose each other. The transmission loss allocations of the generators vary with the loading sequence if there are counter-flows in a system. Hence, it would be fair and logical to divide the benefit of counter-flow among the responsible generators. The transmission loss allocations for Generators A, B and C are 0.0049, 0.0429 and 0.0026 respectively. These losses are the average of losses obtained from 24 loading sequences.

CHAPTER 6: PARTICPATION IN REACTIVE POWER AND SPINNING RESERVE MARKET: A PRICE TAKER'S DESCISION

6.1 Reactive Power

Transmission network of a power system consists of real and reactive elements. Resistance elements are responsible for loss of energy and reactive elements are responsible for storage of real energy. Although reactive elements do not consume energy, reactive power must be provided to maintain a desirable voltage profile throughout the network. This stored electric energy is commonly known as reactive power. Generators, synchronous condensers, static capacitors etc. usually provide reactive power support in a AC system. Transmission lines are also source of reactive power in a power system network. Reactive power in a system can be controlled by either adjusting the excitation of generating units and/or the use of reactive VAR compensators.

Any flow in a transmission line, real or reactive power, produces active and reactive losses. Reactive transmission loss (I^2X) in a system is usually much higher that real transmission (I^2R) loss, since system reactance is generally several times of resistance. Reactive power flow in a power system is a major source of system bus voltage drop. An IPP or a generating agent should be prepared to supply all required reactive power in addition to the real power commitment as a part of its bilateral contract obligation.

A generator under a bilateral contract to supply energy to a special customer may have the ability to supply both real and reactive power including its share of transmission losses. But, in some cases the contracted generator may not be allowed to produce the required amount of reactive power stipulated by the customer contract in order to keep the voltage profile within allowable prescribed limits. In other cases, a contracted generator may not be able to produce the reactive power due to operational limitations. In either case, if generators cannot provide sufficient reactive power due to their operating constraints, system operators use alternative methods to meet the reactive power demand of the full system. A system operator may utilize synchronous condensers, generators, static capacitors etc. to supply the reactive power needed in a given power system

6.2 Allocation of Reactive Power

In a fully deregulated power system network, generators are free to enter into bilateral contracts with their customers. A generator that enters into bilateral contract would be responsible for providing real power as well as reactive power demand of its customer. When a contracted generator would supply a contracted load, there would be transmission losses, both real and reactive, associated with the transaction of power. Transmission losses, both real and reactive, associated with a given bilateral contract can be easily calculated. For real power, a contracted generator might provide losses either from its own sources or from other sources in the network. For reactive power, a generator in a bilateral contract may not be able to provide the required reactive load and the allocated reactive loss either due to its own limited capacity or imposed system constraints. A generator's ability to produce reactive power is limited by its MVA capacity, stability issues and its thermal constraints. It is further limited by its location in the network where a prespecified voltage level must be maintained. Under a bilateral contract, a generator's share of reactive power requirement can be determined by utilizing the ILFA or the MTLA as discussed in Chapters 3 and 4.

In a system, a generator may not be able to produce its share of reactive power loss due to its voltage restriction. The requirements for reactive power in a system may originate from various reasons. A bilaterally contracted generator may not be able to produce its share of reactive power and therefore have to buy reactive power from a third IPP to meet its obligation. In a deregulated electricity market reactive power can be bought and sold along side with real power. An ISO may need reactive power to ensure a predetermined stable voltage profile in order to maintain system security.

6.3 Reactive Power Asking Price – From Supplier's Point of View

A generator's output, both real and reactive powers, is limited by its MVA limit of the generator. In addition, both real and reactive outputs are constrained by their operational maximum limits. Real and reactive power outputs of a generator must also be greater than the minimum required operational levels to insure the stability of the generating units. In between the maximum and the minimum limits of the real and reactive power generation, a generation output is primarily dictated by the maximum MVA limit of the generator. This can be shown in the form of the following equations:

i) if $P<P_{min}$ and $Q=Q_{max}$ then $\frac{dQ}{dP}=0$ (6.1)

ii) if $P=P_{max}$ and $Q<Q_{min}$ then $\frac{dP}{dQ}=0$ (6.2)

iii) if $P_{max}>P>P_{min}$ and $Q_{max}>Q>Q_{min}$ then $\frac{dQ}{dP}=-\frac{P}{Q}$ (6.3)

Where,

P = real power, MW

Q = reactive power, MVAR

$P_{\max}$ = maximum real power producing capacity of the generating unit, MW

$P_{\min}$ = minimum real power producing limit of the generating unit, MW

$Q_{\max}$ = maximum reactive power producing capacity of the generating unit, MVAR

$Q_{\min}$ = minimum reactive power producing limit of the generating unit, MVAR

A model is presented in this work that provides solution to the opportunity cost in a supplier's optimization problem and derives the reservation price vector; the lowest possible prices of real and reactive power that will allow a generator to be operational without losing money.

6.3.1 Reactive Power Asking Price Model

In a competitive market for real power, owners/operators of generators are assumed to be price taker i.e. price of real power, Φ is given. In addition, the total cost of real power production is given by the cost function of a generator,

$$F = aP^2 + bP + c + dQ + e \tag{6.4}$$

The profit function can be written as,

$$\pi = \Phi P + \Psi Q - aP^2 - bP - c - dQ - e \qquad s.t. \qquad Q = f(P) \tag{6.5}$$

Where,

Φ = price of real power, \$/MWHr

Ψ = price of reactive power, \$/MVARHr

π = profit, \$/Hr

a, b, c = cost parameters for real power of a generator

d, e = cost parameters for reactive power of a generator

The IPP-Generator's problem is to maximize profit (π) subject to the operational constraint $S^2 = P^2 + Q^2$. Where S is apparent power (MVA). Reactive power production involves either variable excitation of a generator or other means such as SVC. Hence, the related cost of reactive power can be mainly due to the variation in the excitation of the generator which is usually small compared to the cost of real power.

In the absence of special market for reactive power (ψ=0), the optimization condition is,

$$\Phi = 2aP + b \tag{6.6}$$

which can be interpreted as:

Marginal Benefit from Real Power Production = Out-of-pocket Marginal Cost

With the potential for the existence of a given market for reactive power, at least from the producers' point of view it is possible to derive the minimum asking price.

The minimum asking price will be positive when the owner/operator cannot make positive profit by producing and selling real power only. With positive price of reactive power (ψ>0), using the Lagrangian,

$$L = \Phi P + \Psi Q - aP^2 - bP - c - dQ - e - \lambda\left(P^2 + Q^2 - S^2\right) \tag{6.7}$$

Now maximizing profit with respect to real and reactive power and the Lagrangian multiplier (λ)

$$\frac{\delta L}{\delta P} = \Phi - 2aP - b - 2\lambda P = 0 \tag{6.8}$$

$$\frac{\delta L}{\delta Q} = \Psi - 2\lambda Q - d = 0 \tag{6.9}$$

$$\frac{\delta L}{\delta \lambda} = P^2 + Q^2 - S^2 = 0 \tag{6.10}$$

For the zero profit an IPP or generator owner will operate on the following principle:

Marginal Benefit = Marginal Cost

From Equation (6.9), the following can be obtained,

$$2\lambda = \frac{\Psi - d}{Q} \tag{6.11}$$

Equation (6.8) is rearranged as,

$$\Phi = 2aP + b + 2\lambda P$$

Using Equation (6.11),

$$\Phi = 2aP + b + \frac{P(\Psi - d)}{Q} \tag{6.12}$$

Using Equation (6.3) in Equation (6.12) the following relation can be established,

$$\Phi = (2aP + b) + (-\frac{dQ}{dP})(\Psi - d) \tag{6.13}$$

Where,

$(-\frac{dQ}{dP})$ = Real opportunity cost (in terms of goods)

$(-\frac{dQ}{dP})(\Psi - d)$ = Monetary opportunity cost (in terms of money)

Equation (6.13) can be interpreted as

Marginal Benefit of Real Power Production = Out-of-pocket Marginal Cost + Opportunity Cost in terms of loss of revenue due to a decrease in reactive power production.

Given the price of real power in the market place, the profit maximization rule and zero profit condition are combined to derive the minimum acceptable price of reactive power for a generator.

6.3.2 APPLICATION

The IEEE-RTS (24 Bus) has been used for application for the proposed costing model. It is assumed that there exists a bilateral contract between the generators at bus 7 and load at bus 9. The IPP at bus 9 is contracted to provide the load demand of Customer at bus 9. This implies that the IPP is responsible for supplying both real and reactive power demand of the Customer. While supplying this load there will be associated transmission losses and the IPP is expected to provide this loss too in order to fulfill the contract.

The load at Bus 9 is 240+j106.67. It has been found that Customer's reactive load is 106.67 MVAR and its associated reactive transmission loss is 24.85 MVAR. The IPP is supplying only 44.36 MVAR instead of required 131.52 MVAR due to system constraints. It is obvious that it will be IPP's (connected to Bus 7) interest to find a suitable alternative to provide required remaining reactive power demand for the Customer, unless it has other arrangements with the pool.

The method follows shows the lowest possible prices of reactive power that will allow the other generator to be operational without losing any money. Let us consider the generators connected at Bus 13 as a potential supplier of the reactive power for the bilateral contract between Bus 7 and Bus9. Bus 13 has three 197 MW generators and generator data is given below in Table 6.1. The operating power factor is assumed to be 0.95 for 197 and 350 MW generators and 0.90 for 155 MW generators.

Table 6.1: Generator data at Buses 13 and 23.

Real Power (MW)		Reactive Power (MVAR)		Maximum Apparent Power (MVA)	Cost Parameters		
P_{max}	P'_{max}	Q_{max}	Q'_{max}	S	a	b	c
197	169.12	120	65.00	207.37	0.00300	20.02271	301.22318
350	302.71	210	115.04	368.42	0.00392	8.91965	388.25027
155	140.21	100	75.06	172.22	0.00667	9.27063	206.70340

Where,

P'_{max} = maximum real power producing capacity of the generating unit with Q_{max}, MW

Q'_{max} = maximum reactive power producing capacity of the generating unit with P_{max}, MVAR

Depending on the market price of real power, five possible operating conditions exist, which are discussed below. The reactive power variable cost coefficient d is considered 1% of the corresponding real power cost coefficient b. The reactive power fixed cost coefficient e is considered 10% of the corresponding real power cost coefficient c.

Case I : The price of real power is less than its marginal cost and hence the generator will produce zero real power. But, it will have the full capacity for reactive power production.

$$P = 0\ ;\ Q = Q_{max}\ ;\ \Psi = \frac{c+e}{Q_{max}} + d$$

Case II : The price of real power is such the generator will produce real power (P) within the following limits $P'_{max} > P > 0$.

$$P = \frac{\Phi - b}{2a}\ ,\ Q = Q_{max}\ ;\quad \Psi = \frac{aP^2 + bP + c + e - \Phi P}{Q_{max}} + d$$

Case III : The generator produces fixed amount of real and reactive powers. The prices of real and reactive powers are such that the generator makes up its cost of production.

$P = P'_{max}$, $Q = Q_{max}$;

$$P\Phi + Q\Psi = aP^2 + bP + c + dQ + e$$

$$\text{or, } \Psi = \frac{aP^2 + bP + c + e - \Phi P}{Q} + d$$

Case IV : In this case production of real/reactive power affects the production of reactive/real power. Opportunity cost has a non-zero value for this case.

$P_{max} > P^* > P'_{max}$, $Q_{max} > Q > Q'_{max}$, ; $\frac{dQ}{dP} = -\frac{P}{Q}$

The equation for reactive power price can be written as

$$\Psi = (\Phi - 2aP - b)\frac{Q}{P} + d$$

Case V : The generator produces fixed real and reactive powers. The prices of real and reactive powers are such that the generator makes up it cost of production.

$P = P_{max}$, $Q = Q'_{max}$;

$$P\Phi + Q\Psi = aP^2 + bP + c + dQ + e$$

$$\text{or, } \Psi = \frac{aP^2 + bP + c + e - \Phi P}{Q} + d$$

Case VI : The generator produces maximum real power and zero reactive powers. The price of reactive power is less than its marginal cost and that of real power remains fixed.

$P = P_{\max}, Q = 0$;

$$P\Phi + Q\Psi = aP^2 + bP + c + dQ + e$$

or, $aP^2 + bP + c + e - \Phi P = 0$

Fig. 6.1 shows the graph of reactive power and minimum acceptable price of reactive power for one of the generators connected at Bus 13. This graph can also be called zero profit curve of a generator.

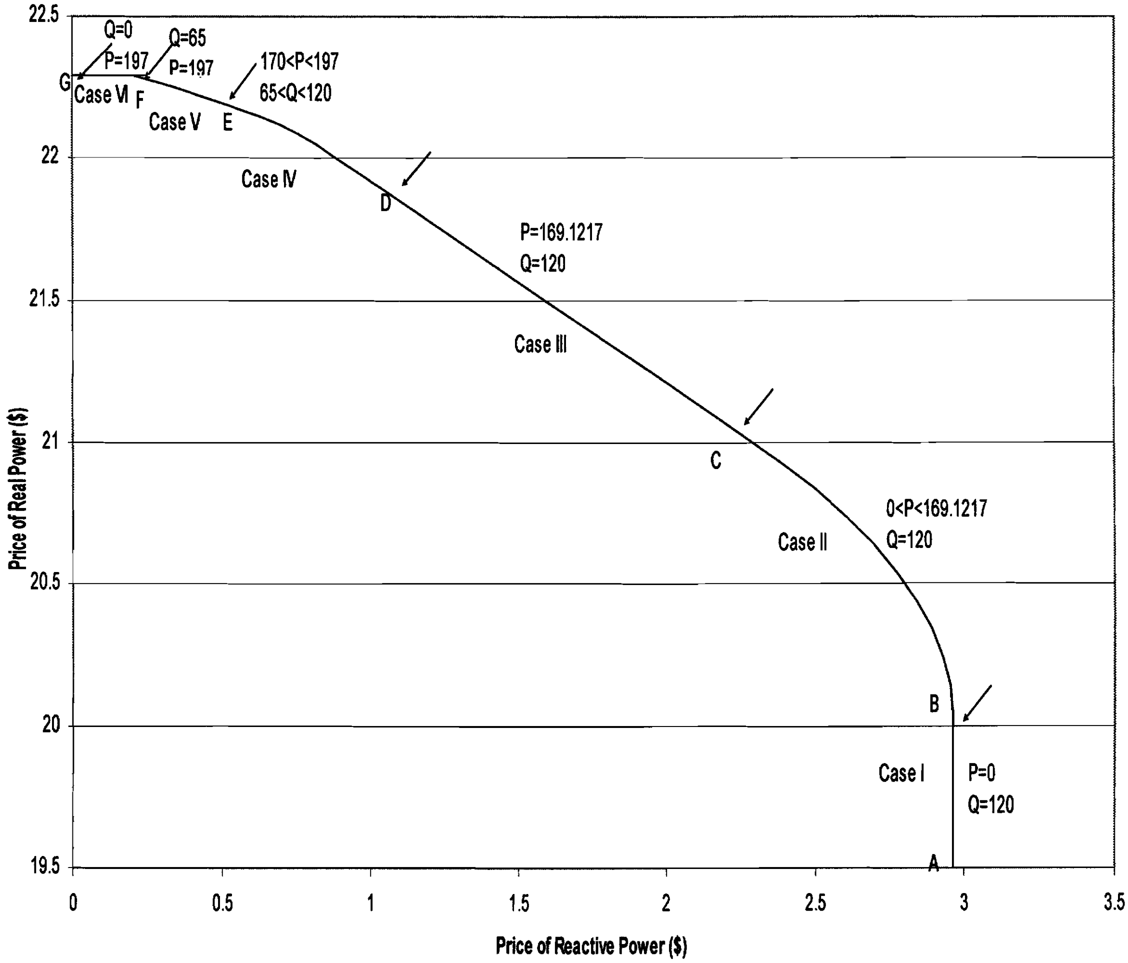

Fig 6.1: Price of real power against the variation of reactive power price for zero profit.

Region A-B in Fig. 6.1 represents the Case I situation. In this region price of real power is less than its marginal cost and hence the generator will produce zero real power. But it will have the full capacity for reactive power production.

The marginal cost of 1st unit or MW power for 197 MW unit

$$= 2*0.003*1+20.02271$$

$$= 20.02871 \quad \$/\text{MWHr}$$

The generator will not produce any real power if price of real power goes below 20.02871 \$/MWHr.

Case II is shown by the region B-C in Fig. 6.1. The generator will produce real power anywhere between 0 and 169.12 MW and 120 MVAR of reactive power.

Case III is depicted by the region C-D in Fig. 6.1. In this region price changes do not affect the production level of a generator and hence a generator will produce real and reactive power (P=169.1217, Q=120) to recover its operating costs.

It can be written as,

$$\Phi P + \Psi Q = aP^2 + bP + c + dQ + e$$

$$169.1217\Phi + 120\Psi = 0.003 * 169.1217^2 + 20.02271 * 169.1217 + 0.01 * 20.02271 * 120$$
$$+ 331.3455$$

$$\Phi = \frac{1}{169.1217}(3831.4540 - 120\Psi)$$
$$\Psi = \frac{1}{120}(3831.4540 - 169.1217\Phi)$$

Region D-E is shown in Fig. 6.1 for Case IV. In this range the production of real and reactive power affect each other and opportunity cost of reactive power has been considered. Reactive power production resides within the range $Q_{min} \leq Q \leq Q_{max}$ for the 197 MW generator.

Finally, Case V is represented by Region E-F in Fig. 6.1. In this region price changes do not affect the production level of a generator and hence a generator will produce real and reactive power (P=197, Q=65) to recover its operating costs.

It can be written as,

$$\Phi P + \Psi Q = aP^2 + bP + c + dQ + e$$

$$197\Phi + 65\Psi = 0.003 * 197^2 + 20.02271 * 197 + 0.01 * 20.02271 * 65 + 331.3455$$

$$\Phi = \frac{1}{197}(4406.2611 - 65\Psi)$$

$$\Psi = \frac{1}{65}(4406.2611 - 197\Phi)$$

Table 6.2 Shows the reactive power production and reactive power price as well as the real power production and its price for the 197 MW generator.

Table 6.2: Price table.

Real Power Price, Φ ($/MWHr)	Real Power, P MW	Reactive Power, Q MVAR	Reactive Power Price, Ψ ($/MVARHr)
19.50000	0.00	120.00	2.961440
20.02271	0.00	120.00	2.961440
20.04000	2.88	120.00	2.961232
20.14000	19.55	120.00	2.951886
20.24000	36.22	120.00	2.928651
20.34000	52.88	120.00	2.891528
20.44000	69.55	120.00	2.840515
20.54000	86.22	120.00	2.775614
20.64000	102.88	120.00	2.696824
20.74000	119.55	120.00	2.604144
20.84000	136.22	120.00	2.497576
20.94000	152.88	120.00	2.377119
21.03744	169.12	120.00	2.246386
21.25000	169.12	120.00	1.946815
21.50000	169.12	120.00	1.594479
21.89000	169.12	120.00	1.044833
22.00990	170.00	118.75	0.875856
22.04725	175.00	111.25	0.819775

22.08281	180.00	102.97	0.760885
22.11648	185.00	93.69	0.698426
22.14821	190.00	83.08	0.631151
22.17797	195.00	70.55	0.556690
22.18930	197.00	65.00	0.523871
22.25000	197.00	65.00	0.339146
22.30000	197.00	65.00	0.187046
22.35000	197.00	65.00	0.034946
22.36149	197.00	65.00	0.000000

6.3.3 Supply Curve for Reactive Power

Three generators at Bus 13 are identical to each other and hence will display similar supply characteristics. In order to show the composite supply curve, Bus 23 has been chosen. Three generators are connected at Bus 23 – two of them are 155 MW generators and one is 350 MW generator. Generators cost parameters are shown in Table 6.1.

6.3.3.1 Construction of Supply Curve

The supply curve for reactive power is constructed based on the price of real power. Any supply curve (reactive power vs. price of reactive power) is based on constant price of real power.

Let, the price of real power, $\Phi = K$

Following steps are to be followed in order to construct a supply curve for reactive power:

1. Draw a line $\Phi = K$. The line crosses zero profit curve of the generator somewhere and get the corresponding price of reactive power from the point of intersection. Let it call Ψ_{min}.
2. Reactive power supply will increase with the increase of Ψ until the generator reaches its Q_{max}. If Ψ_{min} already corresponds to Q_{max} then supply curve will be straight line with $Q = Q_{\max}$.

3. If Ψ_{min} does not corresponds to Q_{max} then find the reactive power price (Ψ*) at which reactive power supply will reach its maximum limit. Beyond the price Ψ*, the reactive power supply stays at its maximum level.

   Now using the following equation, calculate Ψ*

   $$\Phi = 2aP + b + \left(-\frac{dQ}{dP}\right)\Psi^*$$

   Where,

   $P = P'_{\max}$ (maximum real power producing capacity of the generating unit with $Q_{\max}$)

   $Q = Q_{\max}$

   $$\frac{dQ}{dP} = -\frac{P}{Q}$$

4. Now for the portion between the prices Ψ_{min} and Ψ* following of the two approaches can be used.

   a) Linear Approximation: connect the points Ψ_{min} and Ψ* and reactive power supply within this range can be calculated using the straight-line equation.

   b) Exact Method: for any value of reactive power price Ψ ($\Psi_{\min} \rangle \Psi \rangle \Psi^*$) use the following equations to solve for reactive power Q.

   $$\Phi = 2aP + b + \left(-\frac{dQ}{dP}\right)\Psi^*$$

   $$P^2 + Q^2 = S^2$$

Fig. 6.2 shows the composite supply curve for two generators – one 155 MW and one 350 MW for Φ = 13.3788 \$/MWHr. The third generator (155 MW) can be easily added to this composite supply curve. In this case the total supply curve will be lifted up by 100 MVAR. Individual price curves for both generators were plotted first in order to obtain the supply curve. These individual price curves are omitted to save space because they look like Fig. 6.2. The constant value of Φ = 13.3788 \$/MWHr is randomly chosen from the price curve of 155 MW generator.

The portion of the supply curve between Ψ_{min} and Ψ^* is very narrow and the maximum error between exact and the approximate method is found to be 0.18%. Fig. 6.3 shows the exact and the approximate method for this range of the supply curve ($\Psi = 0.013177$ to $\Psi = 0.05965$). The approximate method can be employed to find this portion of the supply curve as it differs very slightly from the exact method.

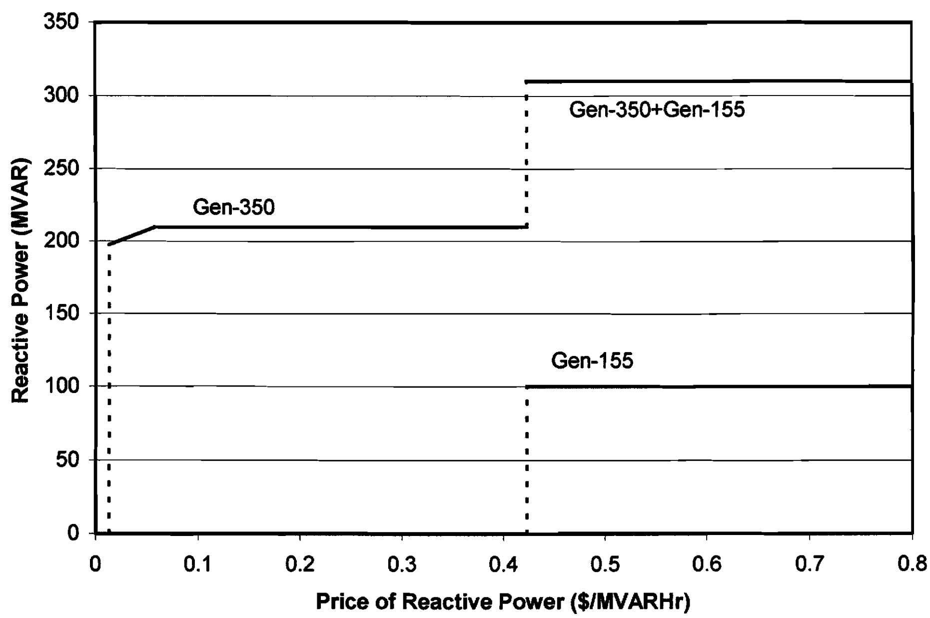

Fig 6.2: Supply curve of reactive power for two different generators.

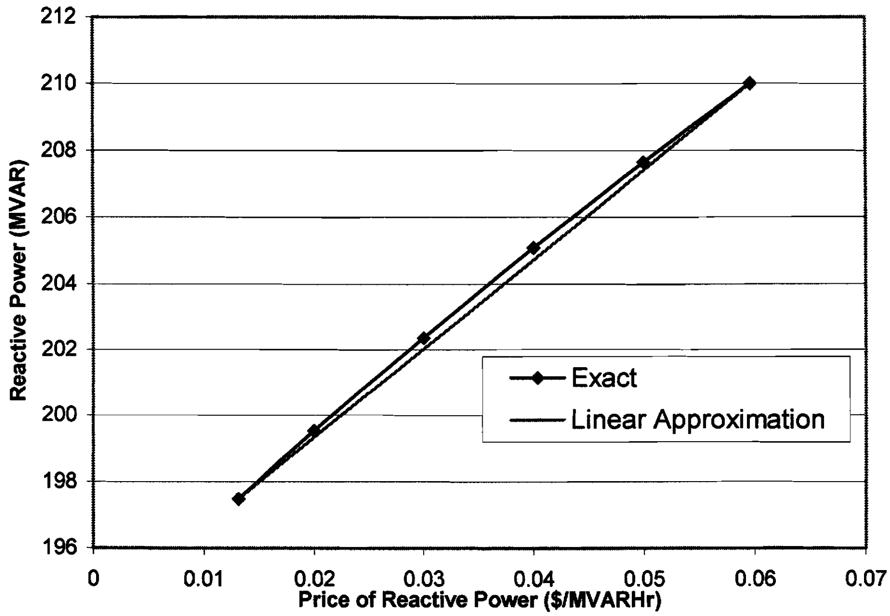

Fig 6.3: Exact and approximate supply curve of reactive power for 350 MW generators for the range $\Psi = 0.013177$ to $\Psi = 0.05965$.

6.4 Spinning Reserve in a Power System

A reliable power system should have higher generating capacity than the actual load demand. The required generating capacity spinning at any given duration in a power system depends on the availability of generating units and the load pattern. Generation must be available in sufficient quantity to account for any unplanned or forced outages as well as normal maintenance of units.

A system prepares a schedule of generation typically for 24 hours based on forecasted load. Scheduled generation at any given time is usually higher than system load. The additional generation beyond system load makes the system capable of handling any sudden unforeseen load changes and possible outages of generating facilities or other facilities.

In practice, all power system components, like any other equipment, have some likelihood of failure. This likelihood can be reduced significantly by proper design and good maintenance practices, but it can never be reduced to zero. Unscheduled shutdown of one or more generating unit or units is one of the sources of power supply failure. In order to overcome this type of situation, additional generating capacity are kept spinning all the time so that they can supply power to the grid at anytime. This additional capacity is known as spinning reserve. A power system network with higher spinning reserve offers higher reliability of supply to its customers. But maintaining this high spinning reserve means higher costs which is eventually paid by its customers. Utilities therefore maintain an optimum level of spinning reserve by striking a compromise between reliability and cost.

Basically two major techniques are used in conventional power system to establish the spinning reserve requirements [58,78,79]. These techniques are:

- Deterministic Approach
- Probabilistic Approach

Deterministic assessment of the spinning reserve can be done using

- Percentage of system load or operating capacity
- Fixed capacity margin
- Largest contingency
- Any combination of the above methods

Different utilities have their own standards and rationale for selecting a particular method. Deterministic approach does not consider the likelihood of any component failure i.e. the probability of failure of generating units, transmission lines etc., in the assessment of spinning reserve. Utilities in the past settled on their reserve requirement with the help of deterministic approach.

A probabilistic approach can recognize the stochastic nature of all system major components and incorporate them in a consistent evaluation of the spinning reserve requirement. The actual magnitude and even the type of risk can be defined as the probability that the system will fail to meet the load or just be able to meet the load for a specified time period. In recent years many utilities started to determine their operating reserve requirements using the probabilistic approaches.

6.5 Spinning Reserve Market

In a deregulated market, energy suppliers compete among themselves to supply energy and, in general, not responsible for the spinning reserve aspect of the energy. The power providers submit their 24-hour bids for selling energy from which successful bidders are chosen by an Independent System Operator (ISO). An Independent System Operator (ISO) controls the operation of a power system network. An ISO, among other things, determines the required amount of spinning reserve required to maintain a reliable supply of electricity. The demand of spinning reserve in a deregulated market has created a market where independent suppliers can bid for offering their generation reserve. The Power Pool, Independent Market Operator or ISO accepts bids for real power as well as spinning reserve. The market of spinning reserve works in the same way as it works for real power. The market operators determine the price of electricity from the data obtained for supply and demand of spinning reserve. The market price is set from the supply and demand relation in such a way that the spinning reserve demand would be satisfied. All suppliers who bid less than the market price are considered as successful bidders and will be considered for supplying the demand. All successful bidders will get paid the market price irrespective of their bidding prices for the amount of committed spinning reserve.

In addition to the bidding price, the response time of a generator is also considered. Given a demand, a generating unit can pick up load at a rate no more than its response rate generally known as ramp rate. The ramp rate essentially dictates the amount of load that an unit can pick up within a given time interval. The response rate of a generator therefore, limits the spinning reserve that a generator can commit and has created two different markets for spinning reserve. They are known as Ten-Minute and Thirty-Minute Spinning Reserve Market.

6.5.1 Ten-Minute and Thirty-Minute Spinning Reserve Market

Ten-Minute Spinning Reserve (TMSR) is defined by the New England ISO [77] as "the Operable Capability of a Generator that is unloaded, is in excess of the quantity required to serve current demand, is able to begin immediately to supply energy to serve demand, is fully available within ten minutes and is able to be sustained for a period equal to the longer of thirty minutes or published NERC or NPCC requirements". Ten-Minute Spinning Reserve Market is served by the generators which are synchronized to the system and can meet the increased load within ten minutes. The New England manual defines the resources for the Ten-Minute Spinning Reserve Market as "the Kilowatts of Operable Capability of an electric Generator or Generators that are synchronized to the system, unloaded during all or part of the hour, and capable of providing contingency protection by loading to supply Energy immediately on demand, increasing the Energy output over no more than ten minutes to the full amount of generating capacity so designated, and sustaining such Energy output for so long as the ISO determines in accordance with market operation rules is necessary to satisfy the immediate contingency". TMSR is a synchronized capacity and can supply power within ten minutes for at least thirty minutes to overcome the initial emergency situation. Thirty-Minute Spinning Reserve is non-synchronized and it is able to begin supply within thirty minutes and lasts for a least sixty minutes.

Ten-Minute Spinning Reserve bids are submitted by the suppliers 24-hour ahead and is evaluated in the same manner as it is done for real power market.

The Thirty-Minute Spinning Reserve Market works in the same way with the difference in the supply time which is thirty minutes instead of ten minutes.

6.6 Supplier's Profit Maximization in a Deregulated Power System – A Composite View

In a deregulated environment market force plays a vital role in determining the supply and demand of a commodity. Electric power characteristics make it different from other commodities but nonetheless it is subject to market force in a deregulated environment. In a competitive market the suppliers are the price takers and they will supply their product, if they can make profit, at a price determined by the market force. In a stable market, price of a commodity sets to a value such that the economic profits of all suppliers become zero. In a volatile market situation, suppliers may make some positive economic profit. If high prices prevail for long time then other suppliers will enter the market and they will increase the supply which will eventually force the price to settle to a lower value where the economic profits of all suppliers become zero. The opposite scenario, where price becomes low, will force some of the suppliers to go out of the business. The reduction in supply, due to the fact that some supplier will cease to exist in the market, will increase the commodity price to such a level where the economic profits of all suppliers become zero again.

In a deregulated electricity market, the suppliers bid for selling their commodity, which is essentially electric power. An Independent Power Producer (IPP) can sell three commodities – real power, spinning reserve and reactive power. According to the principle of Economics, an IPP will try to maximize its profit based on the given market prices. In the following section a profit maximization model is shown for a single supplier in a deregulated power system network.

6.6.1 Profit Maximization Model

In a healthy competitive market an IPP will be a price taker. An IPP will determine the production level of these commodities according to the given market prices of the commodities in such a way that its profit becomes maximum. The profit function of an IPP can be written as:

$$\pi = \Phi P + \Phi_1 T + \Psi Q - aP^2 - bP - c - dQ - e \tag{6.14}$$

Where,

P = real power production. MW

T = spinning reserve, MW

Q = reactive power production, MVAR

Φ = price of real power, \$/MWHr

Φ_1 = price of spinning reserve, \$/MWHr

Ψ = price of reactive power, \$/MVARHr

π = profit, \$/Hr

d, e = cost parameters of reactive power

Usually the production cost of reactive power is very small and varies between 0.5% to 2.5% depending on the machine size. In the present model, cost of reactive power has been included. The profit function is subject to the following operating constraints:

$$(P+T)^2+Q^2 \le S^2 \tag{6.15}$$

$$P_{\min} \le (P+T) \le P_{\max} \tag{6.16}$$

$$Q_{\min} \le Q \le Q_{\max} \tag{6.17}$$

The first inequality constraint can be converted into an equality constrain using a slack variable as follows:

$$(P+T)^2+Q^2+\tau^2 = S^2 \tag{6.18}$$

A generator's output level cannot be increased instantaneously. The rate of change of a generator output is constrained by its ramp-up rate. Usually ramp-up rates of thermal generators varies between 1%-3% MW/minute of its capacity. This means a 100 MW generator can supply only 10 MW in ten minutes for emergency supply considering the ramp-up rate as 1%. Hence the maximum spinning reserve that this generator should commit in a TMSR market is limited to 10 MW. The maximum magnitude of spinning reserve that a generator should commit is constrained by:

$$T \le (\text{ramp rate}) \times \zeta \times P_{\max} \tag{6.19}$$

ζ is either 10 or 30 minutes depending on the type of spinning reserve market: consider the ramp rate to be 2% of the capacity (MW/minute) of a generator. The constraint can be expressed as:

$$T \le \eta \times 10 \times P_{\max}$$

or

$$T \le \eta \times 30 \times P_{\max}$$

where, η = ramp rate of generator, MW/min

The Lagrangian can be expressed as:

$$L = \pi - \lambda\left((P+T)^2+Q^2+\tau^2-S^2\right) \tag{6.20}$$

Taking the derivatives of the Lagrangian with respect to real and reactive power, spinning reserve, the slack variable τ and the Lagrangian multiplier λ:

$$\frac{\partial L}{\partial P} = \Phi - 2aP - b - 2\lambda\ (P+T) = 0 \tag{6.21}$$

$$\frac{\partial L}{\partial T} = \Phi_1 - 2\lambda(P+T) = 0 \tag{6.22}$$

$$\frac{\partial L}{\partial Q} = \Psi - 2\lambda Q - d = 0 \tag{6.23}$$

$$\frac{\partial L}{\partial \tau} = -2\lambda\tau = 0 \tag{6.24}$$

$$\frac{\partial L}{\partial \lambda} = (P+T)^2 + Q^2 + \tau^2 - S^2 = 0 \tag{6.25}$$

From Equation (6.22), the following can be written

$$2\lambda(P+T) = \Phi_1$$

From Equation (6.21), real power production can be determined as:

$$P = \frac{\Phi - \Phi_1 - b}{2a} \tag{6.26}$$

From Equation (6.23) the following can be written:

$$2\lambda = \frac{\Psi - d}{Q}$$

From Equation (6.24)

$$\lambda\tau = 0$$

If λ is not equal to zero, τ must be equal to zero.

$$\tau = 0 \tag{6.27}$$

The interpretation of $\tau = 0$ is given later in the section. Equation (6.22) can be rearranged as follows:

$$\Phi_1 - \frac{\Psi - d}{Q}(P + T) = 0$$

$$(P + T) = \frac{\Phi_1 Q}{\Psi - d} \tag{6.28}$$

Using Equation (6.25), the production level of reactive power can be obtained in the following manner.

$$\left(\frac{\Phi_1 Q}{\Psi - d}\right)^2 + Q^2 - S^2 = 0$$

$$\Phi_1{}^2 Q^2 + (\Psi - d)^2 Q^2 = (\Psi - d)^2 S^2$$

$$Q = \pm \frac{(\Psi - d)S}{\sqrt{\Phi_1{}^2 + (\Psi - d)^2}} \tag{6.29}$$

Finally the optimum level of spinning reserve for an individual supplier can be calculated from Equation (6.28):

$$P + T = \frac{\Phi_1 Q}{\Psi - d}$$

$$T = \frac{\Phi_1 Q}{\Psi - d} - P$$

$$T = \frac{\Phi_1 S}{\sqrt{\Phi_1{}^2 + (\Psi - d)^2}} - \frac{\Phi - \Phi_1 - b}{2a} \tag{6.30}$$

Equations (6.26), (6.29) and (6.30) can be used to evaluate the optimum production level of real power, reactive power and spinning reserve, respectively, of an Independent Power Producer based on given market prices of these commodities. Successive relaxation technique is used to ensure that the constraints (6.16), (6.17) and (6.19) are not violated. If the optimal solutions obtained using Equations (6.26), (6.29) and (6.30) does not satisfy the constraints (6.16), (6.17) and (6.19) then modified equations are used for solutions in the boundary regions of P, Q and T (Appendix-B).

In a competitive market every supplier will produce an optimum combination of real, reactive power and spinning reserve so that their profit is maximized. The positive price of real and reactive power and spinning reserve will motivate a supplier to use all of its available room for production. The combination production levels of P, T and Q will be such that an IPP will utilize its maximum capacity of S. If the IPP leave any room and combined production level is less than S then it will lose money. This fact forces the first inequality constraint into an equality constraint as follows:

$$(P+T)^2 + Q^2 = S^2$$

It has been already found that τ=0 and this indicates that the supplier will utilize its production capacity as long as the prices are more than marginal cost.

The 197 MW generator in Section 6.3.2 is considered for the profit maximization model. The reactive power variable cost coefficient d is considered 1% of the corresponding real power cost coefficient b. The reactive power fixed cost coefficient e is considered 10% of the corresponding real power cost coefficient c.

Figures 6.4-6.6 show the maximum profit as a function of real power, reactive power and spinning reserve prices. Ten-Minute Spinning Reserve bids are considered for the calculation of profit. Figure 6.4 shows variation in profit with changes in reactive power and spinning reserve prices while real power price remains constant at 23 $/MWHr. Figure 6.5 shows variation in profit with changes in reactive power and real power prices while spinning reserve price remains constant at 2 $/MWHr. Figure 6.6 shows variation in profit with changes in real power and spinning reserve prices while reactive power price remains constant 0.6 $/MVARHr. Equation (6.14) indicates that profit is directly proportional to all three prices, price of real power, price of reactive power and price of spinning reserve. Profit will increase linearly with the increase in any one of these three prices. It can be seen from Figures 6.4-6.6 that profit as expected increases as the price of each of the commodities goes up and price of real power has significant effects on profits. Profit rises more sharply as the price of real power increases.

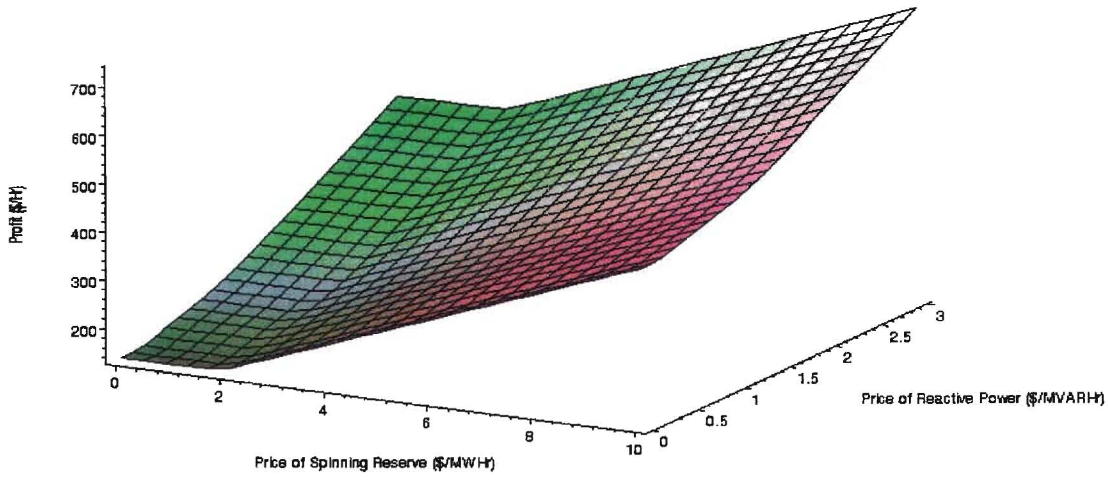

Fig. 6.4: Variation of maximum profit as a function of reactive power and spinning reserve price (real power price is fixed at 23 $/MWHr).

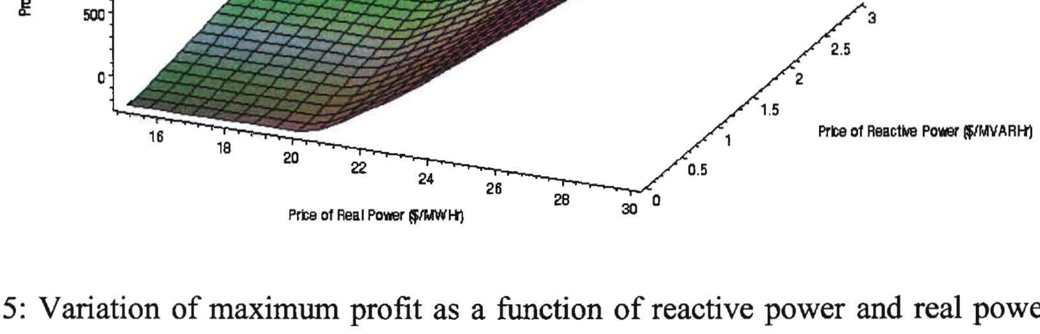

Fig. 6.5: Variation of maximum profit as a function of reactive power and real power price (spinning reserve price is fixed at 2 $/MWHr).

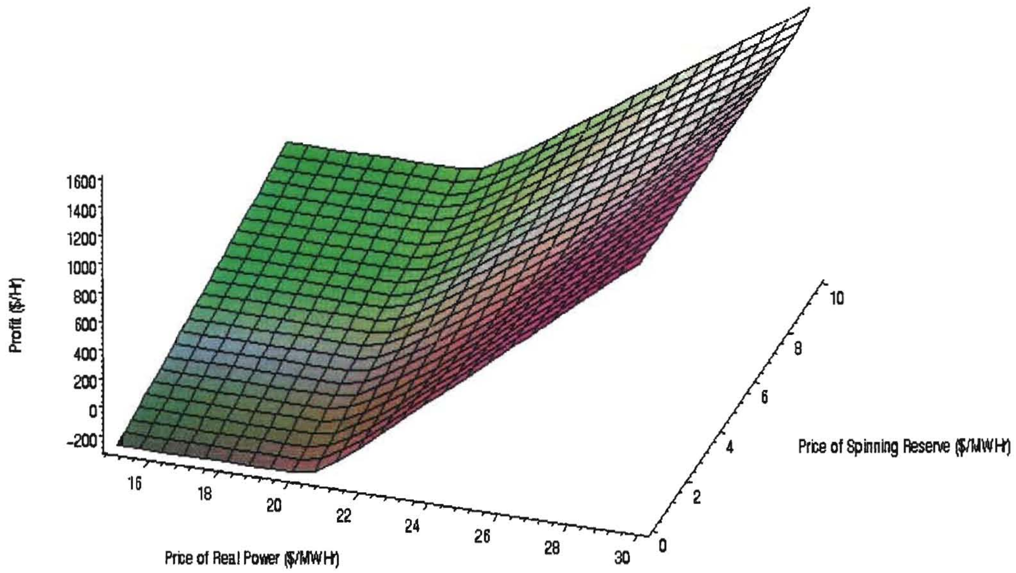

Fig. 6.6: Variation of maximum profit as a function of spinning reserve and real power price (reactive power price is fixed at 0.6 \$/MVARHr).

Figures 6.7-6.15 shows the optimum production level of real power, reactive power and ten-minute spinning reserve. Figures 6.7-6.9 shows changes in real power, reactive power and spinning reserve productions with changes in reactive power and spinning reserve prices while real power price remains constant at 23 \$/MWHr. It can be seen from Figure 6.7 that real power production has negative relation with prices of reactive power and spinning reserve. Optimum level of real power decreases as reactive power and spinning reserve prices go up. Real power production equals to its maximum capacity if prices of reactive power and spinning reserve are relatively small. Consider the price of spinning reserve only, real power production level starts to go down as the price of spinning reserve exceeds \$ 1.6 /MWHr. Maintaining spinning reserve becomes lucrative when the price of spinning reserve becomes higher than 1.6 \$/MWHr. At 2.5 \$/MWHr the spinning reserve reaches its maximum limit and afterwards real power production level remains constant along the axis of price of spinning reserve. Maximum spinning reserve is limited by the ramp-rate of a generator. Now consider the price of reactive power, real power production level remains constant as the price of reactive power increases from zero until reaches 0.9 \$/MVARHr and at this point it starts to decline. Level of real power reaches a constant lower level when reactive power price hits 1.61 \$/MVARHr and when the price of spinning reserve is lower than 1.6 \$/MWHr. Similar effect on the real power production level can seen from Figure 6.7 when both prices of reactive power and spinning reserve are on the higher end. Real power production level reaches its minimum level of 129 MW when price of reactive power is higher than 1.76 \$/MVARHr and price of spinning reserve is higher than 2.5 \$/MWHr.

Figure 6.8 shows that spinning reserve price has little impact on reactive power production. Price range of spinning reserve between 1.7 \$/MWHr to 2.43 \$/MWHr forces the reactive power production level to a lower value. Reactive power price has considerable effect on its production level as it is expected. Reactive power production level starts to rise from 0.12 \$/MVARHr and continues the trend until the price reaches 0.26 \$/MVARHr. Finally, the reactive power reaches its maximum level of 120 MVAR when its price hits 1.72 \$/MVARHr. Spinning reserve production level does not vary much with the price of reactive power as it does with the price of spinning reserve. Figure 6.9 shows that spinning reserve remains at zero until its price reaches 1.65 \$/MWHr where it becomes attractive to maintain a reserve. The production level of spinning reserve remains constant at its maximum level after its price reaches 2.1 \$/MWHr. This is due to the fact that spinning reserve is limited by ramp-rate of a generator and even if its price goes up available amount of spinning reserve is capped by its limit which is shown in Figure 6.9.

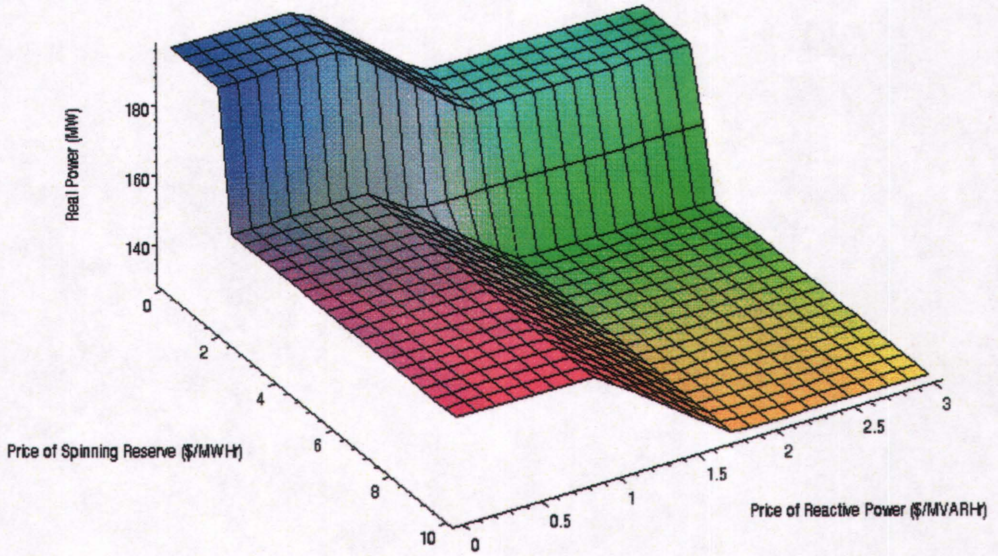

Fig. 6.7: Optimum production of real power as a function of reactive power and spinning reserve prices (real power price is fixed at 23 $/MWHr).

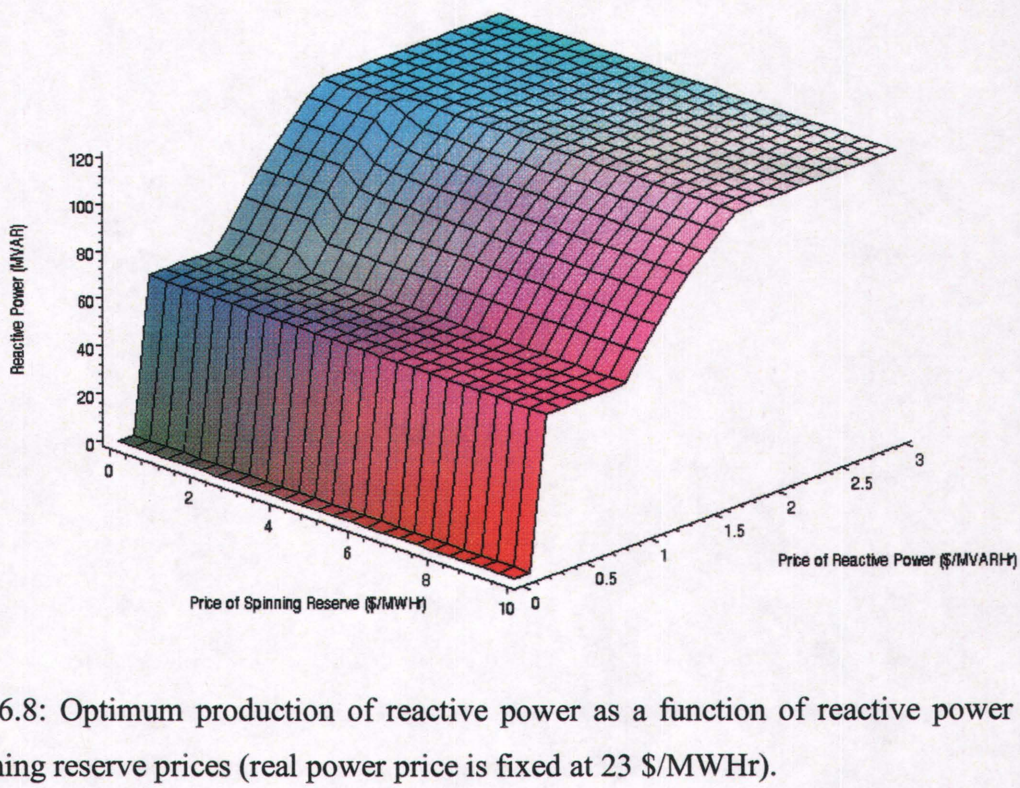

Fig. 6.8: Optimum production of reactive power as a function of reactive power and spinning reserve prices (real power price is fixed at 23 $/MWHr).

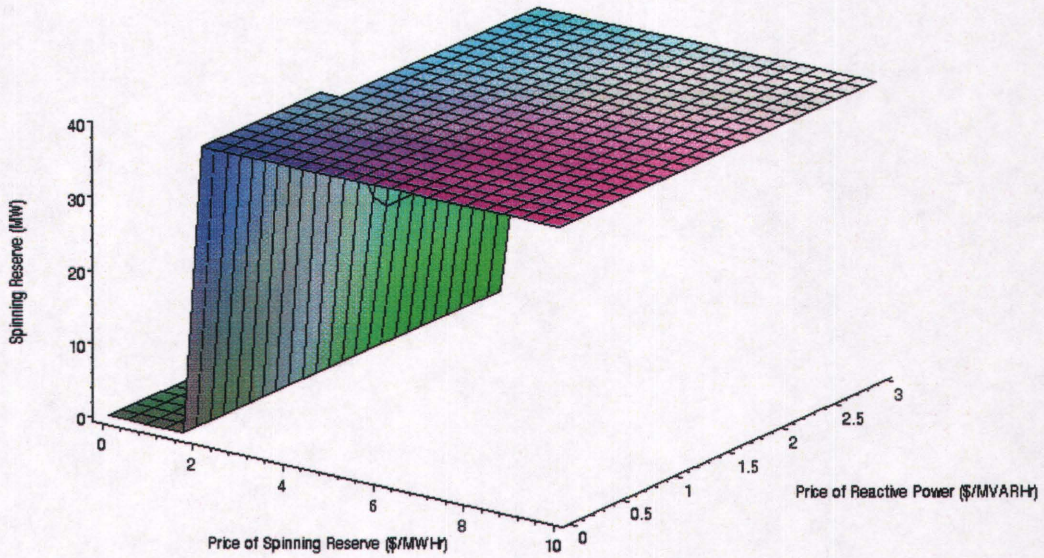

Fig. 6.9: Optimum production of spinning reserve as a function of reactive power and spinning reserve prices (real power price is fixed at 23 $/MWHr).

Figures 6.10-6.12 shows changes in real power, reactive power and spinning reserve productions with changes in reactive power and spinning reserve prices while spinning reserve price is fixed at 2 $/MWHr. Figure 6.10 shows that real power production level remains at zero before its price reaches 20.03 $/MWHr. The increase in price of reactive power tries to reduce real power production level. But the high price of real power, beyond 26 $/MWHr, inspires the producer to maintain the real power at its maximum level of 197 MW. Figure 6.11 shows that reactive power production kicks in as its price reaches 0.12 $/MVARHr. The price of real power at and beyond 20.03 $/MWHr forces the reduction in reactive power. It is evident that production of real power becomes attractive when its price exceeds 20.03 $/MWHr. The higher price of reactive power drags this effect along the axis of price of reactive power. Figure 6.12 shows that spinning reserve maintains its maximum level until the price of real power reaches 22.03 $/MWHr. Spinning reserve production level becomes zero when price of real power reaches 23.7 $/MWHr.

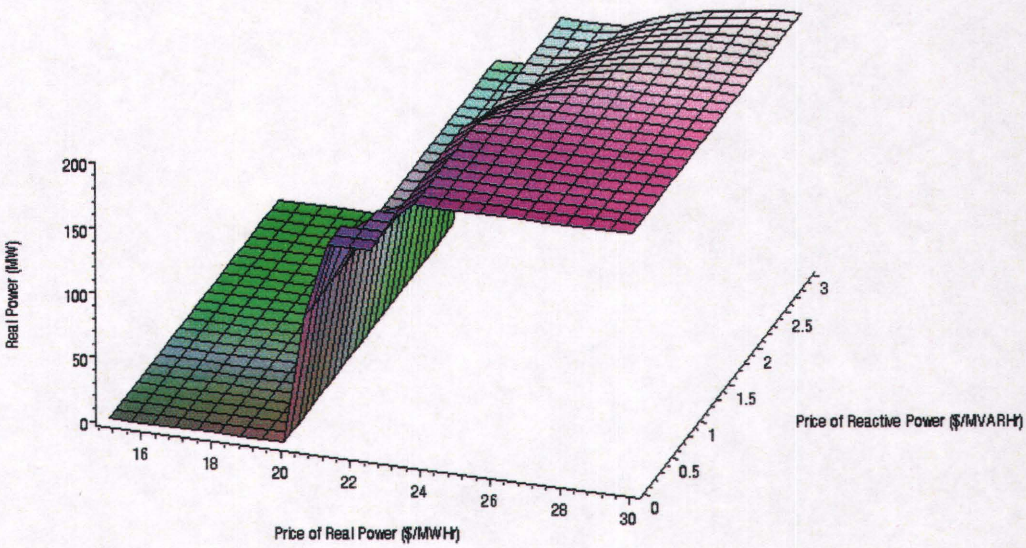

Fig. 6.10: Optimum production of real power as a function of reactive power and real power prices (spinning reserve price is fixed at 2 $/MWHr).

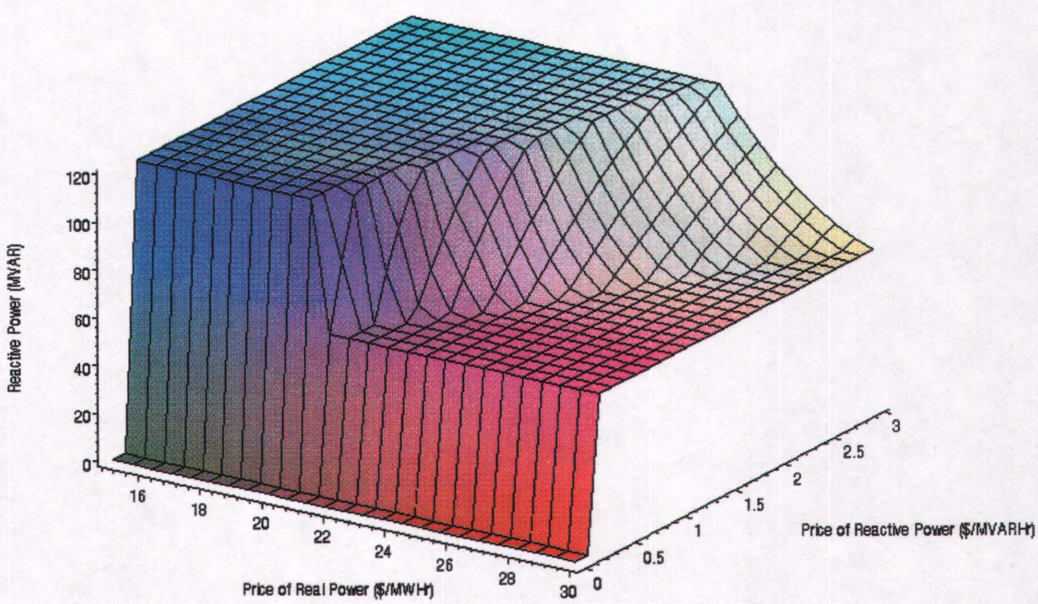

Fig. 6.11: Optimum production of reactive power as a function of reactive power and real power prices (spinning reserve price is fixed at 2 $/MWHr).

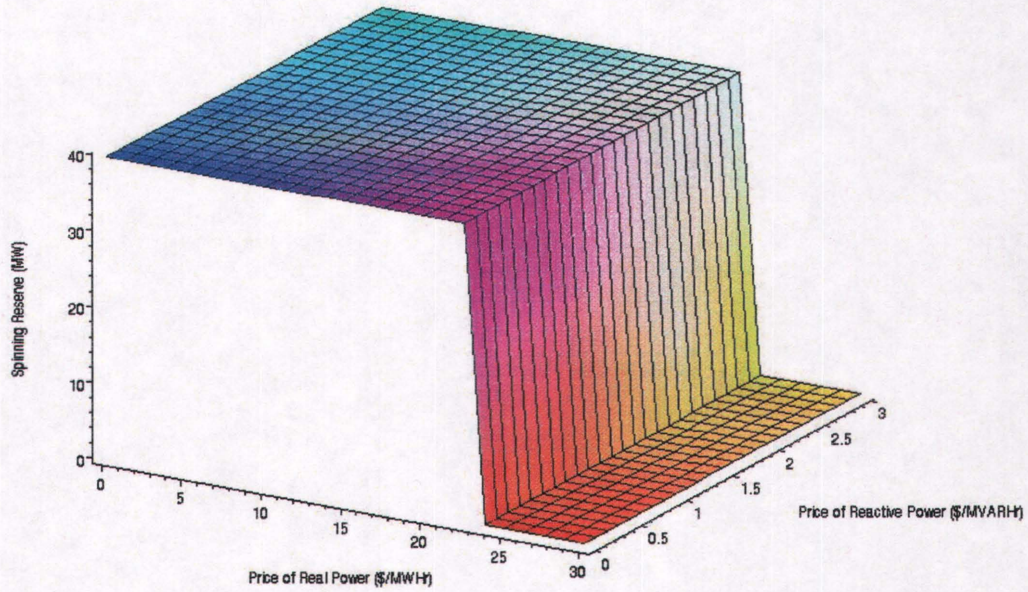

Fig. 6.12: Optimum production of spinning reserve as a function of reactive power and real power prices (spinning reserve price is fixed at 2 $/MWHr).

Figures 6.13-6.15 shows changes in real power, reactive power and spinning reserve productions with changes in reactive power and spinning reserve prices while reactive power price is fixed at 0.6 $/MVARHr. From Figure 6.13 it can be seen that real power production level remains at zero as expected until its price reaches 20.03 $/MWHr. Higher price of real power pushes its production to a higher level and reaches its maximum when the price hits 22.5 $/MWHr. Real power remains at its maximum level until the price of spinning reserve becomes 9.2 $/MWHr and real power becomes 157 MW. Reactive power production level remains at its maximum until real power price reaches 21 $/MWHr and it becomes constant at 65 MVAR when real power price hits 22.5 $/MWHr as it is shown in Figure 6.14. Figure 6.15 shows that spinning reserve level starts to decrease as the price of real power reaches 20.03 $/MWHr. The price of real power at and beyond 21.1 $/MWHr forces the spinning reserve to become zero. It is evident that production of real power becomes attractive when its price exceeds 20.03 $/MWHr. The higher price of spinning reserve drags this effect along the axis of price of spinning reserve.

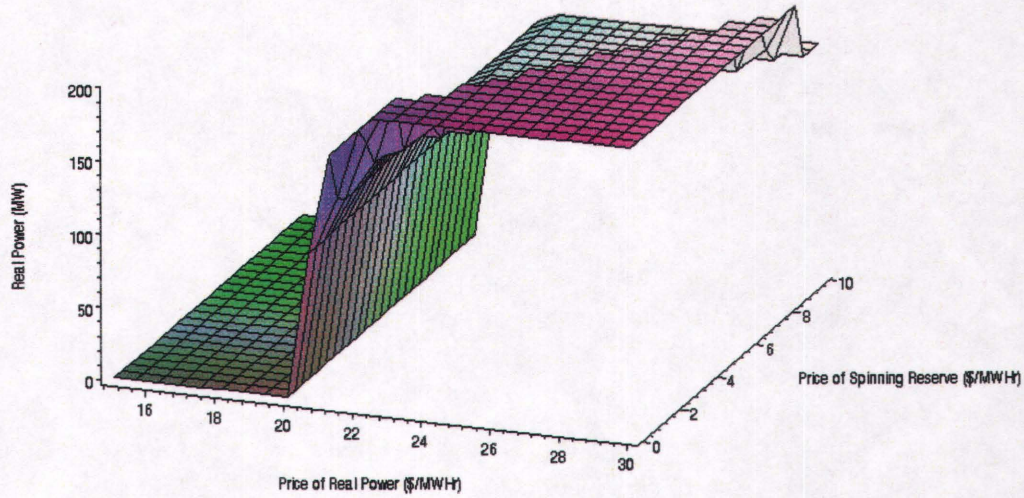

Fig. 6.13: Optimum production of real power as a function of spinning reserve and real power prices (reactive power price is fixed at 0.6 $/MVARHr).

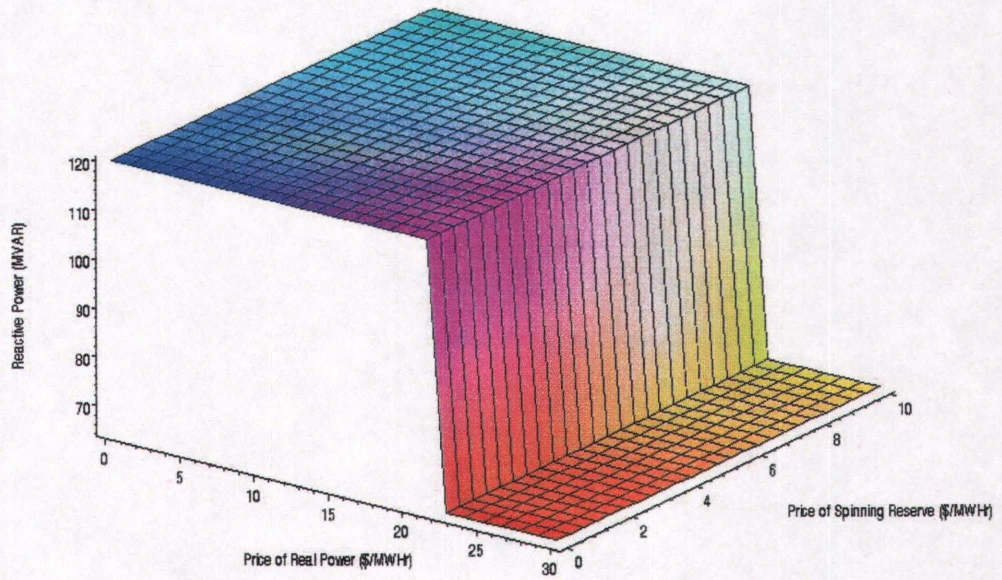

Fig. 6.14: Optimum production of reactive power as a function of spinning reserve and real power prices (reactive power price is fixed at 0.6 $/MVARHr).

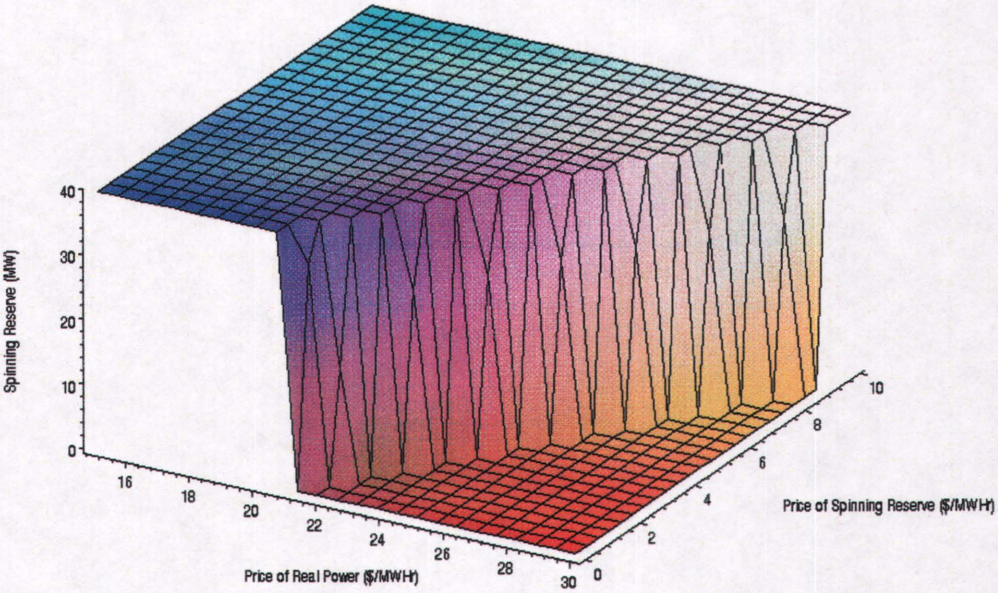

Fig. 6.15: Optimum production of spinning reserve as a function of spinning reserve and real power prices (reactive power price is fixed at 0.6 $/MVARHr).

Thirty-Minute Spinning Reserve bids are also considered and Figures 6.16-6.27 shows results similar to those shown in Figures 6.4-6.15.

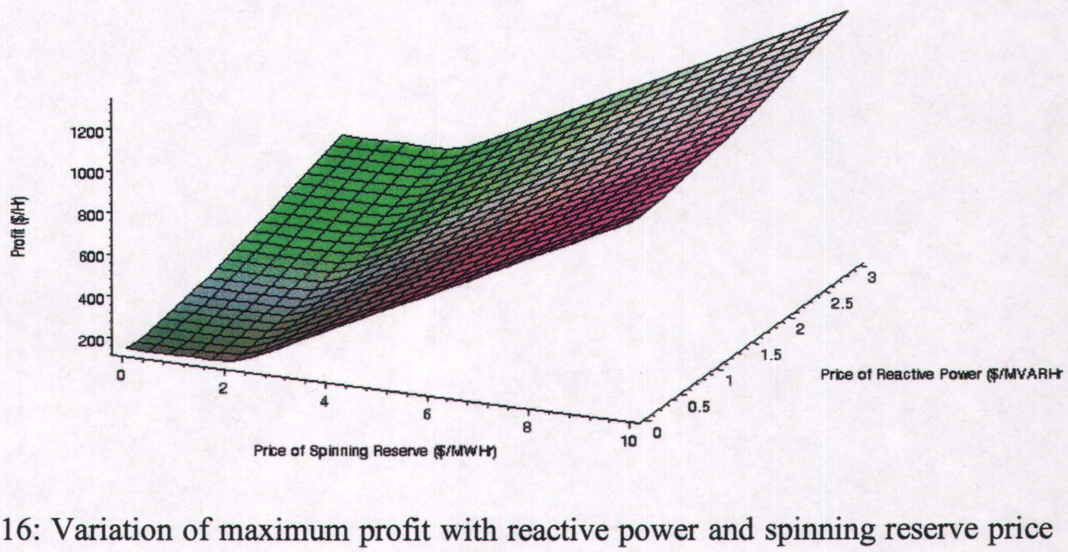

Fig. 6.16: Variation of maximum profit with reactive power and spinning reserve price when real power price is fixed at 23 $/MWHr.

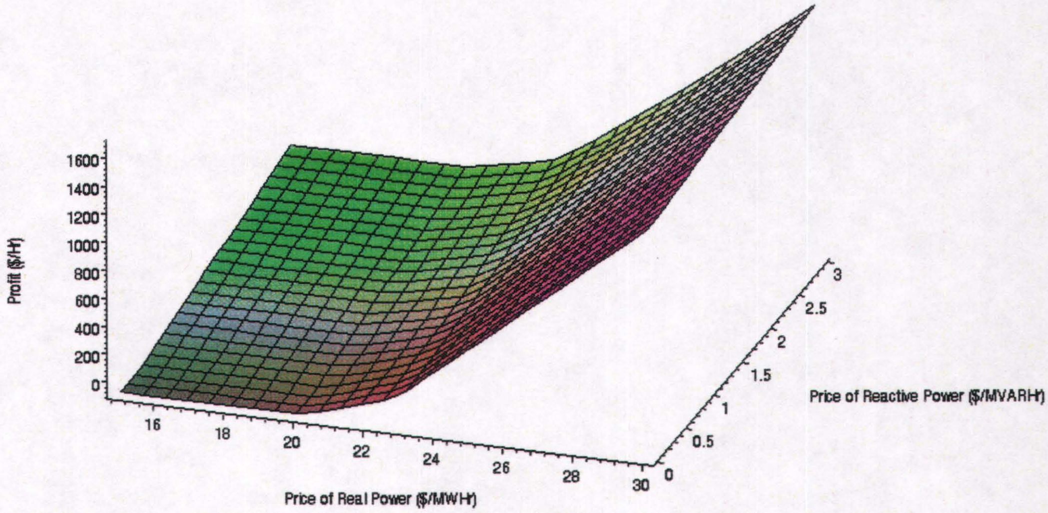

Fig. 6.17: Variation of maximum profit with reactive power and real power price while spinning reserve price is fixed at 2 $/MWHr.

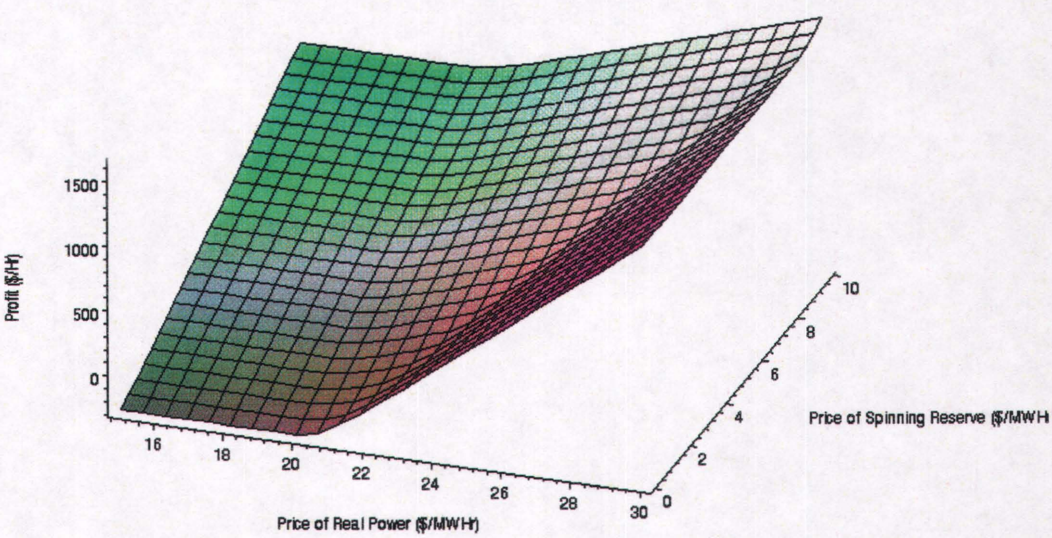

Fig. 6.18: Variation of maximum profit with spinning reserve and real power price while reactive power price is fixed at 0.6 $/MVARHr.

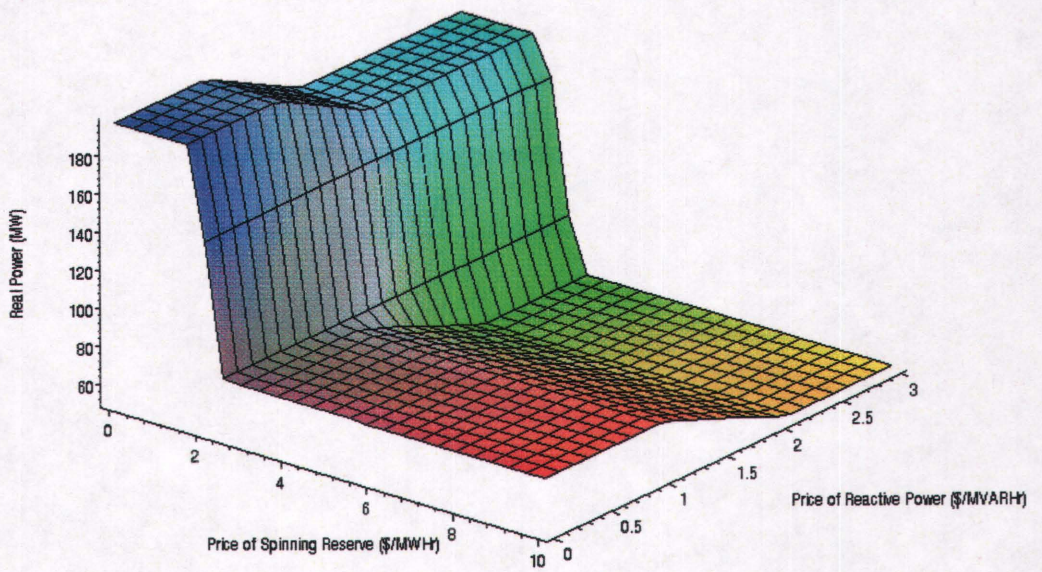

Fig. 6.19: Optimum production of real power as a function of reactive power and spinning reserve prices (real power price is fixed at 23 $/MWHr).

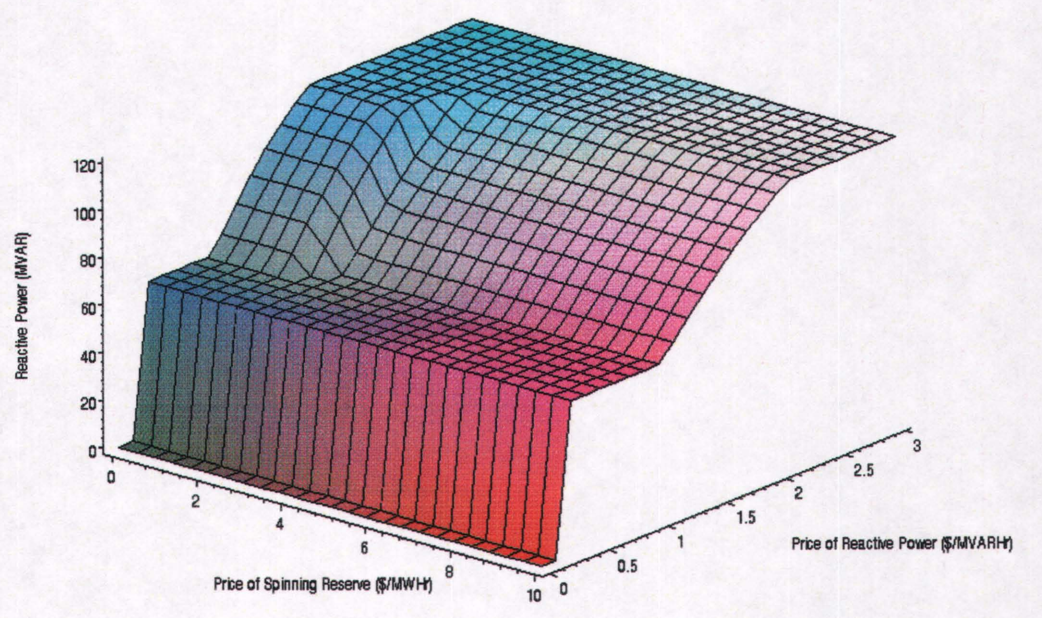

Fig. 6.20: Optimum production of reactive power as a function of reactive power and spinning reserve prices (real power price is fixed at 23 $/MWHr).

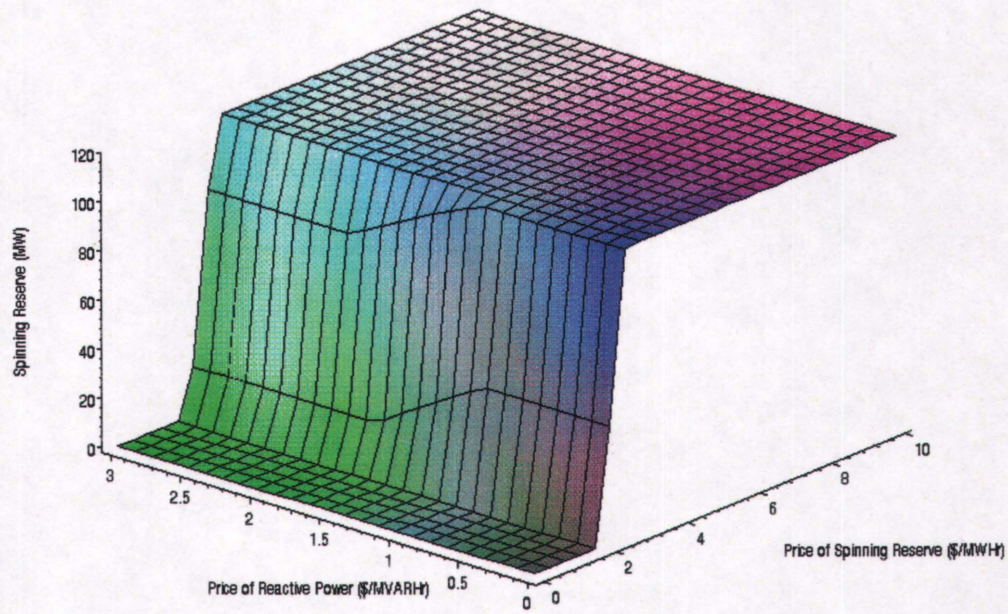

Fig. 6.21: Optimum production of spinning reserve as a function of reactive power and spinning reserve prices (real power price is fixed at 23 $/MWHr).

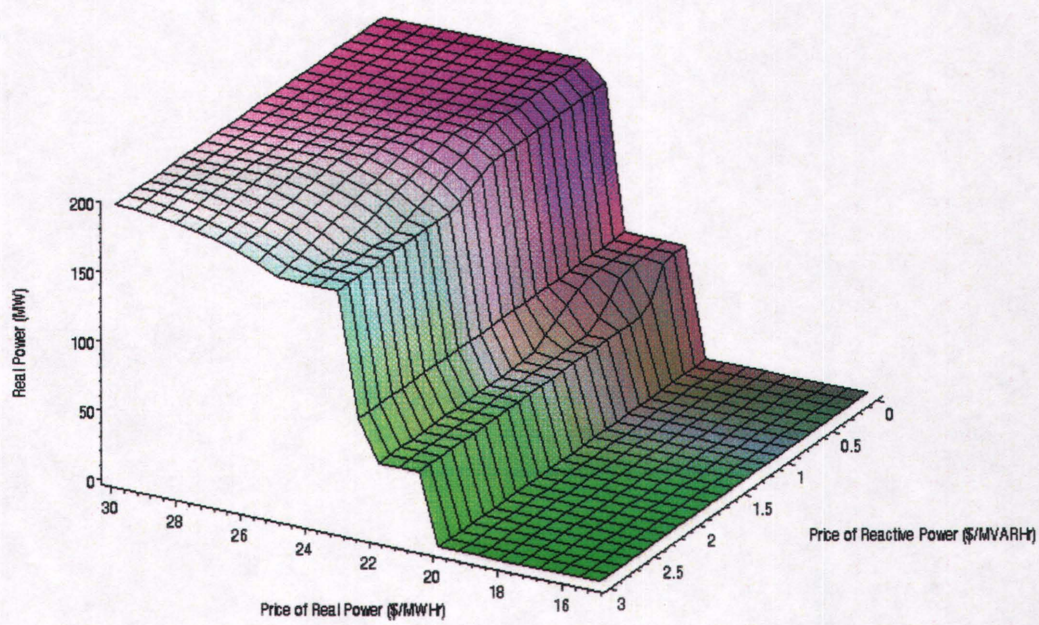

Fig. 6.22: Optimum production of real power as a function of reactive power and real power prices (spinning reserve price is fixed at 2 $/MWHr).

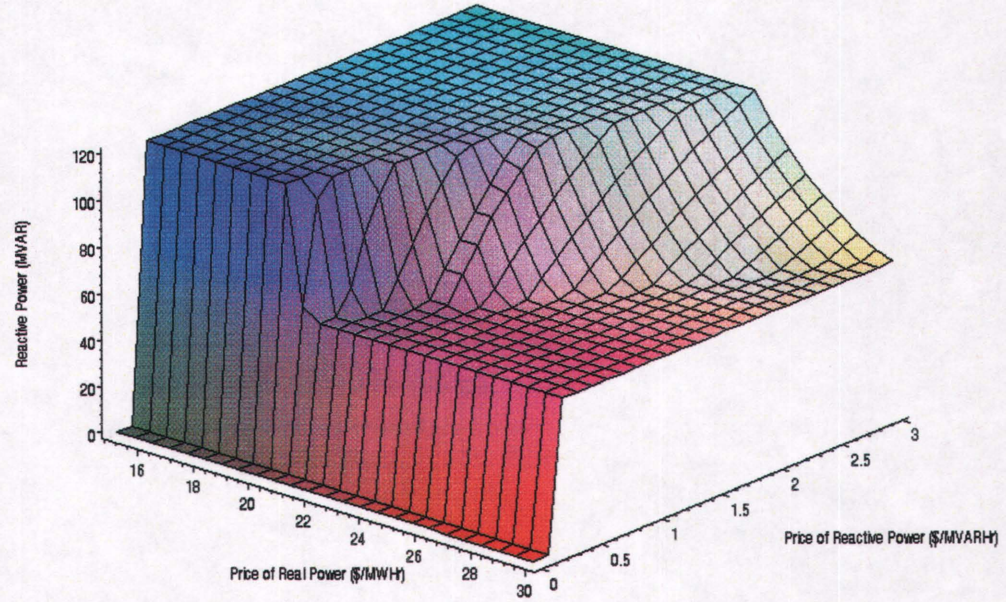

Fig. 6.23: Optimum production of reactive power as a function of reactive power and real power prices (spinning reserve price is fixed at 2 $/MWHr).

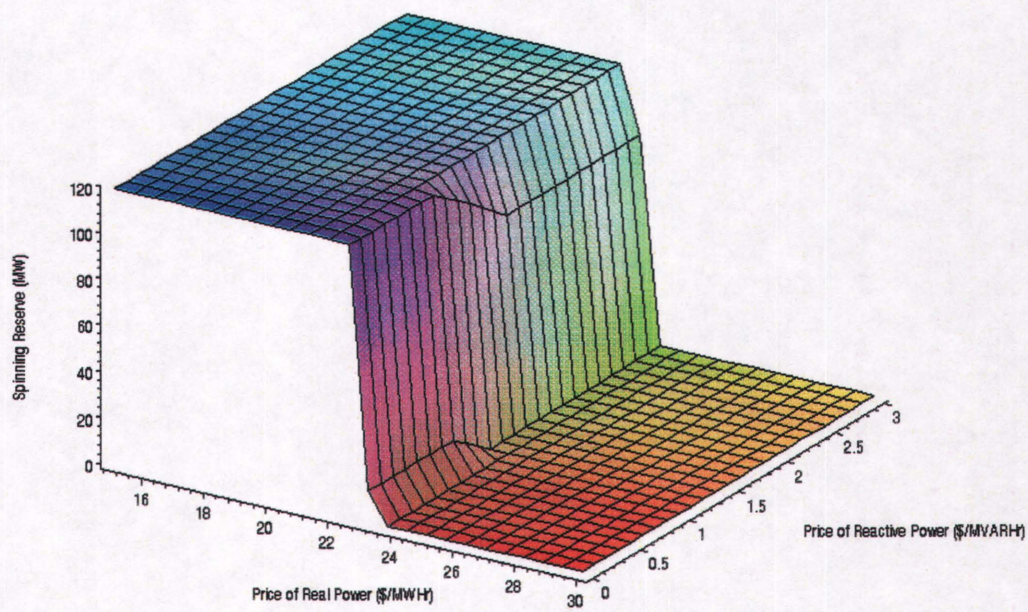

Fig. 6.24: Optimum production of spinning reserve power as a function of reactive power and real power prices (spinning reserve price is fixed at 2 $/MWHr).

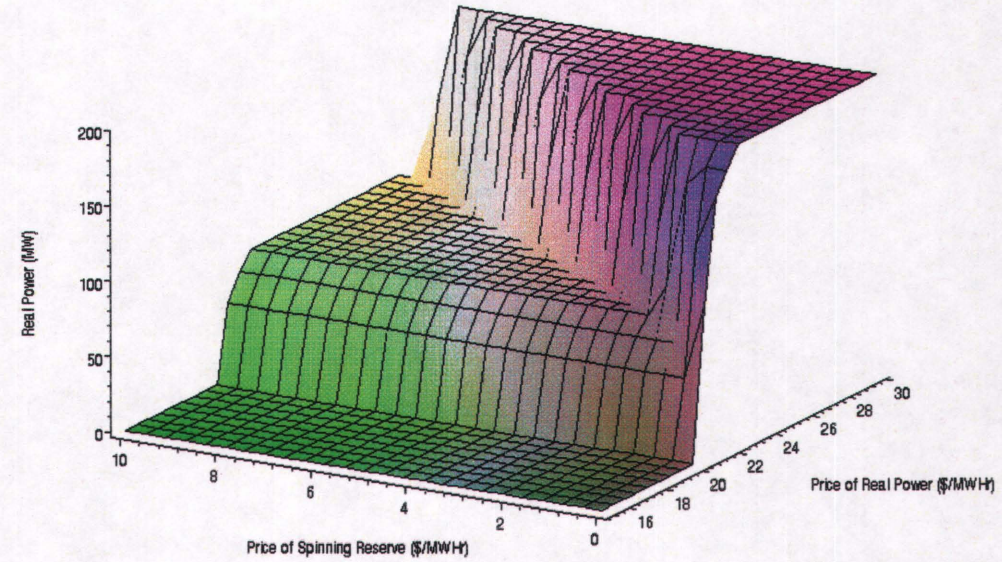

Fig. 6.25: Optimum production of real power as a function of real power and spinning reserve prices (reactive power price is fixed at 0.6 $/MWHr).

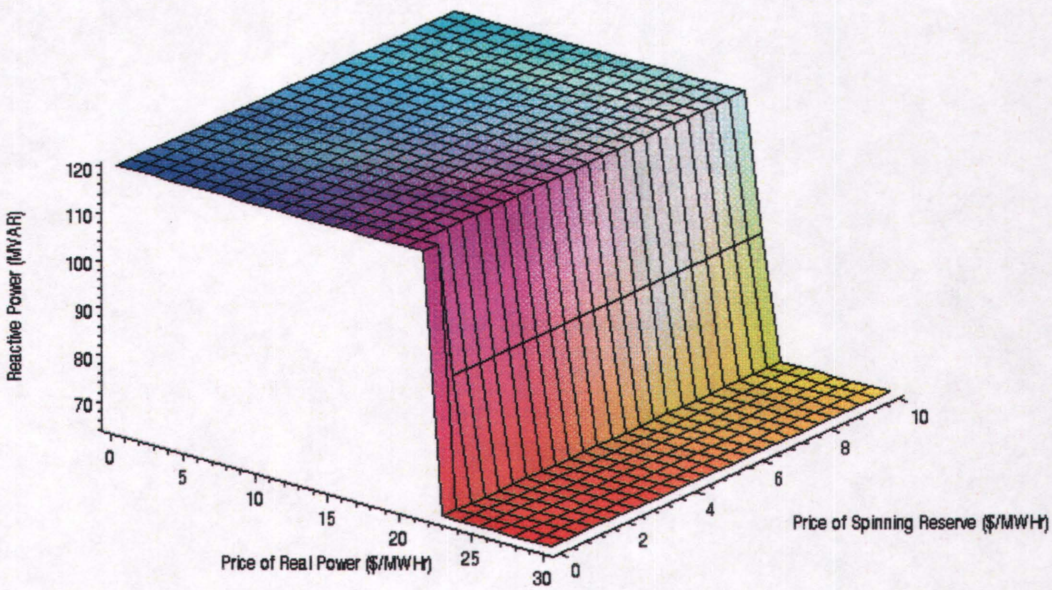

Fig. 6.26: Optimum production of reactive power as a function of real power and spinning reserve prices (reactive power price is fixed at 0.6 $/MWHr).

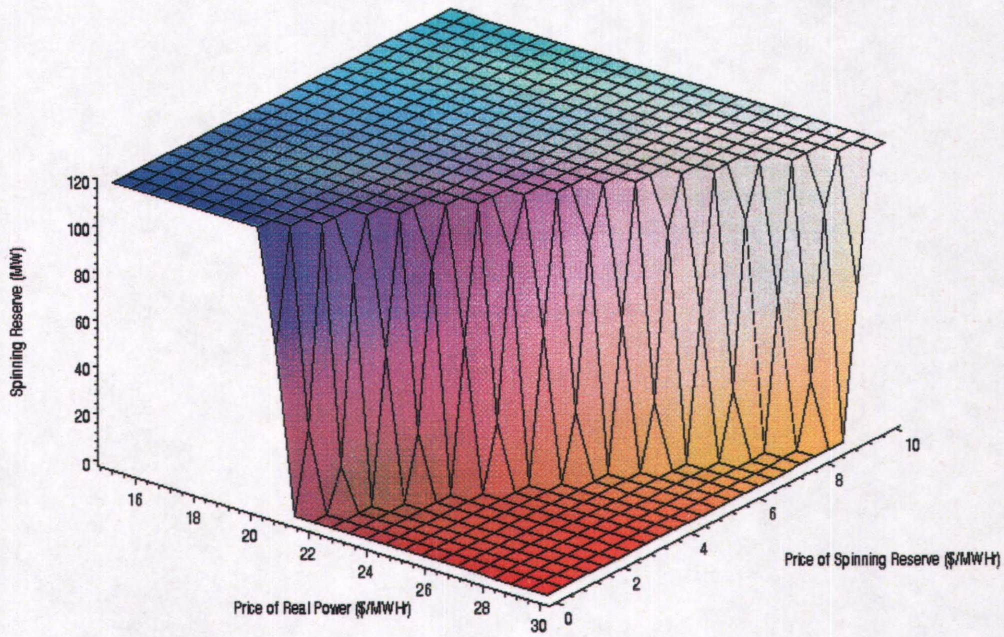

Fig. 6.27: Optimum production of spinning reserve as a function of real power and spinning reserve prices (reactive power price is fixed at 0.6 $/MWHr).

6.6.1.1 Zero Profit Conditions

In an open competitive market the objective of any supplier is to maximize its profit. In a stable market the economic profits of all suppliers in the long run become zero. The market prices determine the number of suppliers and the quantities of commodities. A supplier that can make at least zero profit will survive in the competition and a supplier that cannot reach the minimum zero profit condition will be out of business. A zero profit condition of a supplier indicates the minimum price for its product that the supplier can withstand without losing money.

In a deregulated power market, an IPP will always be interested to know its limits of production so that it can survive in the market. An IPP is a price taker and it will determine its production based on the market prices. It is important for an IPP to know the level of production for a given price so that the profit will be zero. The combination of prices of real and reactive power and spinning reserve plays an important role in a zero profit condition. The zero profit can be obtained from combination of prices of these three commodities. If the price of any one of them goes lower than the corresponding price at zero profit condition an IPP might not be interested to supply power, because doing so it will incur losses.

The minimum acceptable level of prices are obtained by solving the Equations (6.21)-(6.25) and by making the profit function equal to zero in Equation (6.14). The zero profit and maximum profit conditions together provide the minimum production levels for real power, reactive power and spinning reserve for the given price ranges of these three commodities. These production levels are shown in Appendix-C. These price ranges for the 197 MW generator is shown in Figures 6.28 and 6.29 for the Ten-Minute Spinning Reserve and Thirty-Minute Spinning Reserves respectively.

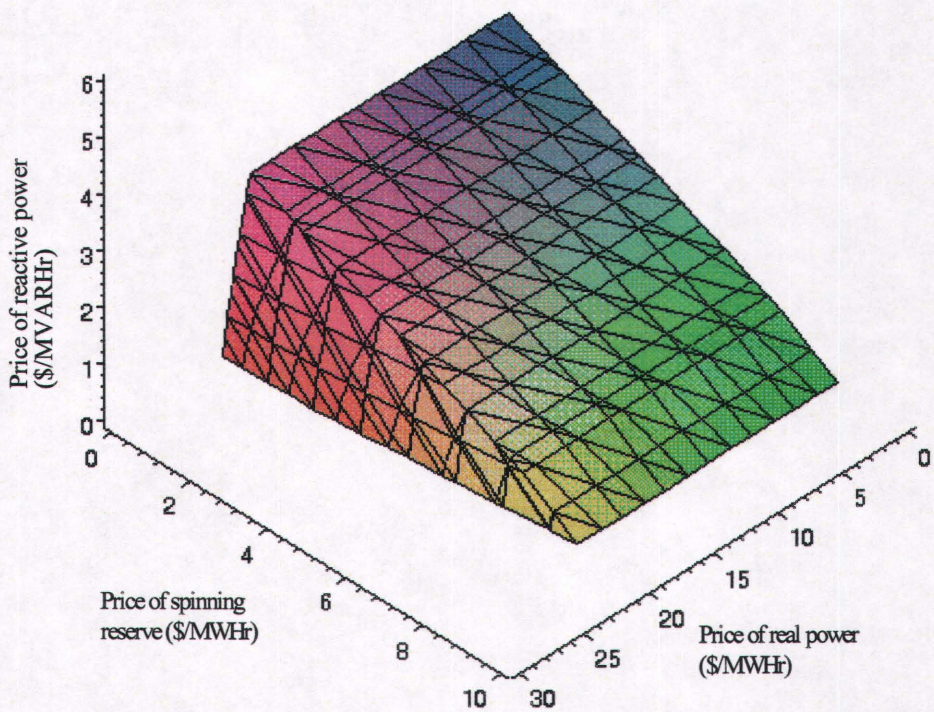

Fig. 6.28: Real power, reactive power and spinning reserve price combinations for zero profit conditions for Ten-Minute Spinning Reserve Market.

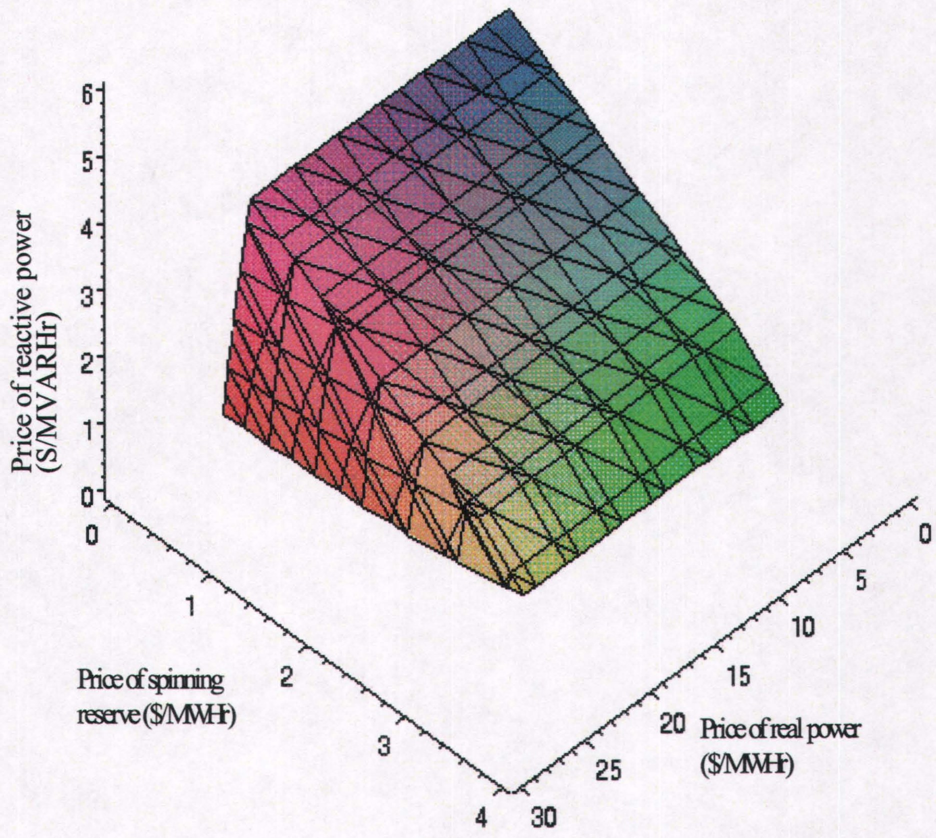

Fig. 6.29: Real power, reactive power and spinning reserve price combinations for zero profit conditions for Thirty-Minute Spinning Reserve Market.

6.6.1.2 Supply Curves

An IPP is likely to have more than one generator and it is also possible to have generators with different capacities. It is possible to construct a supply curve for real power, reactive power and spinning reserve. These supply curves will show the optimum levels of electric power as a function of market price. A supply curve of real power will demonstrate the optimum real power production level with a variation in real power market price. An IPP can utilize this information to decide when to bring additional generation in the bidding process. Similarly supply curves for reactive power and spinning reserve would show the optimum level of production as a function of their respective market prices. Figures 6.30-6.32 show the supply curves for real power, reactive power and spinning reserve respectively. Ten-Minute Spinning Reserve is considered in this case. Figure 6.30 shows the supply curve real power for 155 MW and 350 MW generators as functions of the price of real power. Spinning reserve and reactive power prices are assumed to be constant at 2 \$/MWHr and 0.5 \$/MVARHr respectively. It can be seen from Figure 6.30 that 155 MW generator will bid for supplying real power when real power price is 9 \$/MWHr or higher and 350 MW generator will bid for supplying real power when real power price is 9.3 \$/MWHr or higher. Higher prices of real power would encourage the producer to sell all of its real power as the incentive for maintaining a spinning reserve is comparatively low.

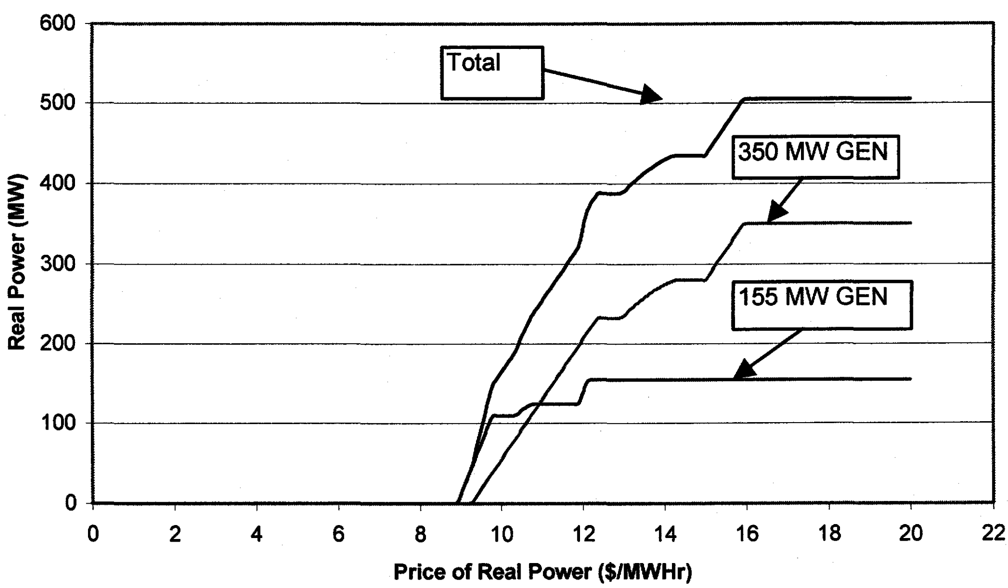

Fig. 6.30: Supply curves for real power for 155 MW and 350 MW generators.

Figure 6.31 shows the supply curves for reactive power for 155 MW and 350 MW generators as functions of the price of reactive power. Spinning reserve and real power prices are assumed to be constant at 2 \$/MWHr and 13 \$/MWHr respectively.

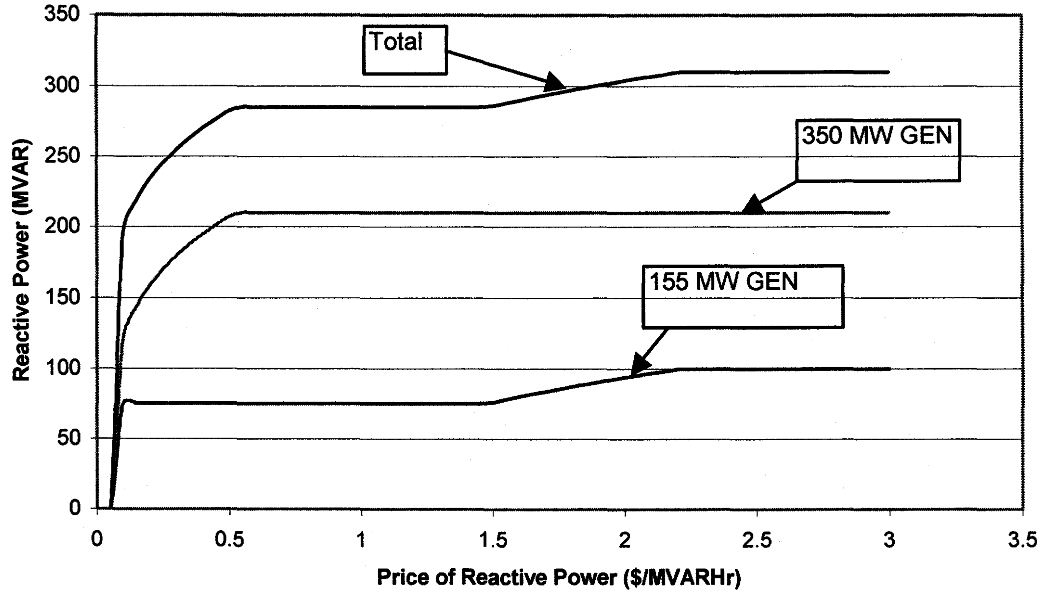

Fig. 6.31: Supply curves for reactive power for 155 MW and 350 MW generators.

Figure 6.32 shows the supply curves for spinning reserve for 155 MW and 350 MW generators as functions of the price of reactive power. Reactive power and real power prices are assumed to be constant at 0.5 $/MVARHr and 13 $/MWHr respectively.

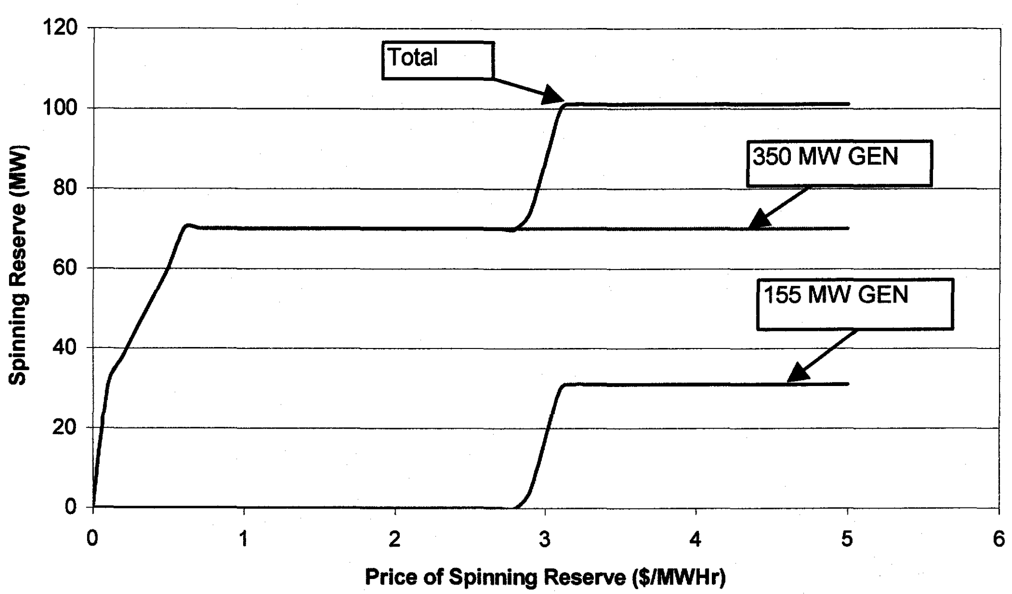

Fig. 6.32: Supply curves for spinning reserve for 155 MW and 350 MW generators.

Figures 6.30-6.32 are the supply curves for real power, reactive power and spinning reserve as functions of their corresponding prices. Two generator are considered for the construction of these supply curves. An IPP may own more than two generators. Any number of generators can be included in these supply curves using the same technique.

6.7 Summary

A single supplier's optimization challenge has been addressed in this chapter. Price of electric power in a deregulated environment is governed by market rules. Market Clearing Price of electric power is determined from the bids obtained from suppliers. After several market iterations an IPP would be able to forecast the market prices. Based on forecasted market prices an IPP will determine the production levels for its commodities – real power, reactive power and spinning reserve.

Two models have been presented in this chapter. The First model determines the minimum acceptable price of reactive power as a function of real power price using the zero profit condition. Results for a 197 MW generator is shown in Section 6.3.2. This model also provides supply curve for reactive power when an IPP owns more than one generator. This model will help an IPP in its decision making process regarding its participation in a deregulated power market. Supply curve from a supplier that owns one 350 MW and two 155 MW generators are shown in Section 6.3.3.1.

The second model includes spinning reserve as a product in addition to real and reactive power. Optimum levels of production of these three commodities are indicated by this model using profit maximization technique. The model also provides the minimum acceptable price ranges for an IPP. This has been obtained by combining maximum profit and zero profit conditions. Maximum profit and minimum acceptable price ranges are shown for 197 MW generator in Sections 6.5.1 and 6.5.1.1. Supply curves for real power, reactive power and spinning reserve as a function of their respective prices are shown for two generators in Section 6.5.1.2.

CHAPTER 7: CONCLUSIONS

7.1 Conclusions

There are many challenges associated with the deregulation of power industry. Some of them are technical which include transmission loss allocations, transmission access usage, spinning reserve, reactive power pricing etc. and the others are non-technical. The non-technical challenges originate from political, socio-economic and environmental issues.

Of all the technical issues related to deregulation, allocation of transmission losses and transmission access are the most contentious. This research work dealt with the issue of transmission loss allocation for bilateral contracts, counter-flow, optimization challenges of generating plants in a deregulated power system. An electric power system must have sufficient supply of power in order to meet the customer's requirement. The market operators determine the price of electricity from the data obtained for supply and demand of real power. This price is commonly known as market clearing price. The market clearing price is set from the supply and demand relationship in such a way that all power demands would be satisfied. Bilateral contracts are considered on the top of this wholesale electric power market.

Under a deregulated structure, power generating utility or an IPP may enter into a bilateral contract with a customer. Bilateral contracts as stated earlier lead to the challenge of sharing transmission losses among the competing generating utilities. Any method used to allocate transmission losses should be viewed by the generating entities as transparent and fair.

In the research work reported in this thesis, methods have been developed to allocate transmission losses in a deregulated system where generators are tied to the loads through bilateral contracts. For a given configuration, a discrete change in a given load will be accompanied by a discrete change in transmission loss. The discrete change in the transmission loss can be attributed to the generator that is contracted to the load provided all other factors remain the same. In a deregulated network with bilateral contracts, a discrete change in the transmission loss can be evaluated by increasing a load by a discrete amount while keeping all other loads at their previous levels. All loads can be changed in a sequential manner and the corresponding discrete change in transmission loss can be aggregated. Each generator will be responsible for supplying its load as well as its share of transmission loss. In short, this is the working principle that has been used in the work reported in this thesis. This working principle has been realized by two different methods.

The Incremental Load Flow Approach (ILFA) is a relatively simple technique. It utilizes conventional AC load flow technique in a successive manner. Some modifications in an AC load flow are required to evaluate transmission loss allocation. The modifications are very simple and can be implemented with little difficulty. The second method, the Marginal Transmission Loss Approach (MTLA) is a direct consequence of mathematical reasoning. If a transmission loss can be expressed as a function of loads then its marginal rates can be evaluated with respect to the loads. Traditionally, transmission losses have been expressed as functions of generation. This is useful for the purpose of economic optimization. In order to allocate transmission loss, we need transmission loss to be expressed as a function of loads. Starting from the expression of bus injection in a network, a convenient and useful expression for transmission loss has been derived which is a function of loads. The MTLA has provided a means to check the results obtained by the ILFA. Although the MTLA uses a different approach, it provides similar loss allocations as obtained by the ILFA.

The MTLA is a complex mathematical method based on general transmission loss formula. This generalized method is applicable for electric power network of any size. The derived equations in Chapter 4 can be used to find the loss allocation between any generator and customer in a power system network. The MTLA can identify loss shares for multiple contracts signed by a single generating utility with different customers or a customer who is getting supply from multiple generators. This method would separate each transaction and determine the transmission loss related to a particular contract signed. Although the formulation of mathematical expression for the MTLA is a complex process, its execution time is much faster than that of the ILFA. The ILFA requires load flow technique that needs iterations to obtain the solution for one load level and this process has to be repeated for every incremental load level. Whereas in the MTLA, a set of simultaneous equations are solved to calculate loss allocations and no iteration is required during its execution.

The formulation of the MTLA is based on some assumptions. Those assumptions have been made to keep the challenge of transmission loss allocation manageable. Although they help to keep the formulation of the MTLA relatively simple, it has been found that the loss allocations obtained by the MTLA differ noticeably in some load situations than those obtained by the ILFA because of the assumptions. The assumptions of constant Z-bus, bus voltages and angles (only three different values used) over the full range of load demand are responsible for the deviations in results to a large extent. In an ideal case, current bus impedance matrix, bus voltages and angles should be used instead of assuming them to be constant. But it is cumbersome to store bus impedance matrix, bus voltages and angles for every incremental load level used in the MTLA and moreover, a load flow program has to be utilized to update those parameters. The use of updated bus impedance matrix, bus voltages and angles in every load level in the MTLA, however, gives fairly accurate and close results to those obtained by the ILFA.

The methods reported in this work consider bilateral contracts between the generators and the customers in a deregulated network for the allocation of transmission losses. Full deregulation would allow bilateral contracts between the generators and the buyers and it is expected that there would be a number of such contracts in a fully deregulated system. A generator in the bilateral contract with its customer is supposed to meet the customer's load along with the associated transmission loss. In some cases, a generator might not be able to produce its share of loss. In such cases, the ILFA or the MTLA would be very useful to determine how much this particular generator would owe to other generators in the system and the liable generator would pay accordingly. In some cases, some generators may have to produce more reactive power than their allocations in order to maintain the minimum voltage level in the system. In such cases, these generators should be compensated by other generators and the ILFA or the MTLA would be a useful tool to determine other generators' liabilities.

The developed methods are applied to different networks in this work. A small test network has been used to calculate transmission loss allocation where a full deregulation has been considered. A full deregulation implies the existence of bilateral contracts only and there will be no power pool in this kind of system. Loads are assumed to be bulk and transmission losses, both real and reactive, are obtained for different bilateral contracts in the test system. The results obtained from two different methods are compared and shown in Tables and graphs. A larger system, developed by IEEE has been used in this work in order to calculate transmission losses in a mixed-mode system. A mixed-mode system consists of both power pool and bilateral contracts. In a power pool, sellers and buyers of power bid for power in hour-ahead, day-ahead market or in a spot market. In the IEEE-RTS system, bilateral contracts are considered over an existing power pool. Two different bilateral contracts are assumed to exist in the IEEE-RTS and the ILFA and the MTLA are applied to calculate transmission losses caused by each transaction.

The methods developed in this work can be used for cost-study of a bilateral contract. Using one of these methods any generator/customer may know in advance the share of transmission losses for its load demand. Once the generator knows its share of transmission loss for supplying a particular load it can consider different options for compensating the loss. The generator might consider supplying both load and loss or buying the loss from other sources or becoming a part of an economic load dispatch. A cost analysis has been done to have an idea about the feasibility of different options. The cost analysis gives an idea about what a generator should do depending on the type of agreement with a customer.

In an electrical power system network, total transmission loss might decrease if an additional generator is brought into the system. This happens as the power flows in some transmission lines from the new generator oppose initial flows in those particular lines. The latter flows are known as counter-flows. It has been mentioned earlier that counter-flow is caused by two opposing flows in a line. Therefore, it needs at least two generators to reduce overall transmission loss and create the existence of counter-flow. Hence it is very important to identify the generators contributing to counter-flow along with the calculation of transmission loss allocation. The developed methods of transmission loss allocation can handle this challenge very effectively. The ILFA and MTLA are based on sequential increment of loads. The sequential load increment has been exploited to identify the existence of counter-flow in the system.

The existence of counter-flow in a system can be revealed by exploiting the loading sequences of bilateral contracts. In presence of a counter-flow the computed transmission loss will change with a change in the loading sequence. Since a counter-flow cannot exists without the presence of two sources whose flows oppose each others'. The resulting benefit should be divided among the sources. When more than two sources with bilateral contracts are involved, transmission losses for all loading sequences are calculated. A generator's share of transmission loss is obtained by averaging the shares of the losses obtained from all loading sequences. This distribution of transmission loss is done in light of sharing the benefit of all and in essence acceptable in absence of a specific sharing formula.

In a deregulated electricity market reactive power can be bought and sold along side with real power. The requirements for reactive power in a system may originate from various reasons. A bilaterally contracted generator may not be able to produce its share of reactive power and therefore have to buy reactive power from a third generator to meet its obligation. An ISO may need reactive power to ensure a predetermined voltage profile in order to maintain system security.

Although the production cost of reactive power is very small when compared to the production cost of real power, a generator may have to limit the production of real power and lose potential business if it wants sell reactive power.

A model has been developed, based on the price of real power and real and reactive power generating capacity of a generator, to calculate the minimum acceptable price of reactive power for a single supplier in a competitive energy market. This model helps a generating entity to find a way to charge other generating entities for providing reactive power support for them. A composite supply curve can be obtained for multiple generating units. The IEEE-RTS has been used for the model developed in Chapter 6.

A power system usually commits generating capacity above that necessary to meet its load demand. The additional capacity, known as spinning reserve makes the system capable of handling unforeseen load changes and possible outages of generating facilities or other facilities. Spinning reserve helps ensure a reliable system operation but at an additional cost. A higher level of reliability can be achieved by increasing the magnitude of spinning reserve. An increase in the spinning reserve will result in a corresponding increase in the operational cost. Utilities decide an optimum level of spinning reserve based on a compromise between reliability worth and reliability cost. An Independent System Operator in a deregulated system determines the required amount of spinning reserve that is needed for maintaining a reliable supply of electricity. The demand for spinning reserve has created a market and generators and independent suppliers can bid for offering their reserve as they bid for selling their real power. The rate at which a generating unit can pick up its load is limited by its ramp-up time or ramp rate. This fact limits the spinning reserve that can be held by a generator and has created two different markets for spinning reserve. They are known as Ten-Minute and Thirty-Minute spinning reserve market.

In a deregulated electricity market, a generator or an Independent Power Producer (IPP) can sell three commodities – real power, spinning reserve and reactive power. Any IPP will try to maximize its profit based on the given market prices. Chapter 6 deals with the profit maximization model for a single supplier in a deregulated power system network. The objective function of the model is to indicate the optimum level of production of three commodities - real power, reactive power and spinning reserve based on the forecasted market price. An IPP will decide its production levels to gain maximum profit. The IEEE-RTS has been utilized as an example system and maximum profit and productions are shown in graphs for given market price variations considering both Ten-Minute and Thirty-Minute Spinning Reserves.

A set of minimum acceptable price vectors have been evaluated and shown in Chapter 6. This set governs the decision of an IPP not to produce anything if market prices go below the minimum acceptable prices. An IPP will incur losses if it continues to produce when any of the prices falls below the minimum acceptable price ranges. Both Ten-Minute and Thirty-Minute Spinning Reserves are considered to obtain the minimum acceptable price vectors.

An IPP usually owns more than one generating units. These generating units may vary in size. Techniques have been developed to construct composite supply curves for an IPP when it owns multiple numbers of units. The supply curves for real power, reactive power and spinning reserve based on the variation in market prices are shown in Chapter 6.

REFERENCES

[1] Viscusi, W.K., Vernon, J.M., Harrington Jr., J.E., "Economics of Regulation and Antitrust", Lexington, Mass, 1992

[2] J.C.Moorhouse, "Competitive Markets for Electricity Generation", The Cato Journal, Vol. No. 3, URL: http://www.cato.org/pubs/journal/cj14n3-3.html.

[3] R.M.Isaac, V.L.Smith, "Natural Monopoly and contested Markets: Some Experimental Results", Journal of Law and Economics 27:91-113.

[4] URL: http://www.ucei.berkeley.edu/ucei/Restructuring_Archive/ab_1890.pdf.

[5] Robert J. Michaels, "Stranded In Sacramento California Tries Legislating Electrical Competition",
URL: http://www.cato.org//pubs/regulation/reg20n2i.html.

[6] URL: http://www.orrick.com/news/elect/electric/4.htm.

[7] URL: http://reason.com/0106/fe.ml.power.shtml.

[8] Thomas Sowell, "The Cause of the California Electricity Shortages: "Price Controls"", January 11, 2001
URL:
http://www.capitalismmagazine.com/2001/january/ts_electricity_shortages.htm.

[9] URL: http://www.energy.ca.gov/electricity/capacity.html.

[10] URL:
http://www.mv.com/ipusers/zeropoint/IEHTML/FEATURE/FEATR/REiss36California.htm.

[11] J.Taylor, P.VanDoren, "California's Electricity Crisis: What's Going On, Who's to Blame, and What to Do", URL: http://www.cato.org/pubs/pas/pa-406es.html.

[12] URL: http://www.powerscorecard.org/state_issues.cfm?state=Pennsylvania.

[13] URL: http://www.energy.gov.ab.ca/electric/restruct/overview.htm.

[14] URL: http://www.ibew.org/stories/01journal/0107/canadaderegulation.htm.

[15] URL: http://www.saskpower.com/html/message.html.

[16] URL: http://www.eia.doe.gov/emeu/pgem/electric/contents.html.

[17] URL: http://www2.ing.puc.cl/power/southamerica/argentina.html.

[18] URL: http://www.ing.puc.cl/power/southamerica/southamerica.htm.

[19] URL: http://www.eia.doe.gov/emeu/cabs/brazil.html.

[20] E. Masud, "Deregulation Fiasco, Red Flag for Developing Countries California Electricity Deregulation Discards Cost Minimization, Favors Profit Maximization", February 5, 2001, URL: http://www.twf.org/News/Y2001/0108-Electric.html.

[21] URL: http://shakti.hypermart.net/conventional.html.

[22] URL: http://www.bd-pdb.org/.

[23] URL: http://www.bd-pdb.org/distribution.htm

[24] D.Shirmohammadi et al., "Transmission Dispatch and Congestion Management in the Emerging Energy Market Structures", IEEE Transactions on Power Systems, Vol. 13, No. 4, November 1998, pp. 1466-1474.

[25] H.P.Rosenof, "A Spot Market for Electricity",
URL: http://www.gensym.com/successstories/powerauc.htm

[26] Z.Alaywan, J. Allen, "California Electric Restructuring; a Broad Description of the Development of the California ISO", IEEE Transactions on Power Systems, Vol. 13, No. 4, November 1998, pp. 1445-1452.

[27] H.Singh, A.Papalexopoulos, "Competitive Procurement of Ancillary Services by an Independent System Operator" ,IEEE Transactions on Power Systems, Vol. 14, No. 2, May 1999, pp. 498-504.

[28] R.Bjorgan, C.Liu, J.Lawarrée, "Financial Risk Management in a Competitive Electricity Market", IEEE Transactions on Power Systems, Vol. 14, No. 4, November 1999, pp. 1285-1291.

[29] R.W.Ferrero, J.F.Rivera, S.M.Shahidehpour, "Application of Games with Incomplete Information for Pricing Electricity in Deregulated Power Pools", IEEE Transactions on Power Systems, Vol. 13, No. 1, February 1998, pp. 184-189.

[30] C.Silva, B.F.Wollenberg, C.Z.Zheng, "Application of Mechanism Design to Electric Power Markets", IEEE Transactions on Power Systems, Vol. 16, No. 4, November 2001, pp. 862-869.

[31] H.H.Happ, "Cost of Wheeling Methodlogies", IEEE Transactions on Power Systems, Vol. 9, No. 1, February 1994, pp. 147-156.

[32] G. Strbac, D. Kirschen, S. Ahmed, "Allocating Transmission System Usage on the Basis of Traceable Contributions of Generators and Loads to Flows", IEEE Transactions on Power Systems, Vol. 13, No. 2, May 1998, pp. 527-534.

[33] R. Allan, D. Kirschen, G. Strbac, "Contributions of Individual Generators to Loads and Flows", IEEE Transactions on Power Systems, Vol. 12, No. 1, February 1997, pp. 52-60.

[34] J. Bialek, "Identification of Source-Sink Connections in Transmission Network", IEE Fourth Conference on Power System Control and Management, London, 16-18 April 1996, pp. 200-204.

[35] J. Bialek, "Topological Generation and Load Distribution Factors for Suppliment Charge Allocation in Transmission Open Access", IEEE Transactions on Power Systems, Vol. 12, No. 3, August 1997, pp. 1185-1193.

[36] Y. Tsukamoto, I. Iyoda, "Allocation of Fixed Transmission Cost to Wheeling Transactions by Cooperative Game Theory", IEEE Transactions on Power Systems, Vol. 11, No. 2, May 1996, pp. 620-627.

[37] F. D. Galiana, M. Ilic′, "A Mathematical Framework for the Analysis and Management of Power Transactions under Open Access", IEEE Transactions on Power Systems, Vol. 13, No. 2, May 1998, pp. 681-687.

[38] F. D. Galiana, M. Phelan, "Allocation of Transmission Losses to Bilateral Contracts in a Competitive Environment", selected for publishing in IEEE Transactions on Power Systems, Vol. 13, No. 2, May 1998, pp. 681-687.

[39] J.W.M.Cheng et al., "Studies of Bilateral Contracts with Respect to Steady-State Security in a Deregulated Environment", IEEE Transactions on Power Systems, Vol. 13, No. 3, August 1998 pp. 1020-1025.

[40] M.D.Anderson, J. Yang, "Tracing the Flow of Power in Transmission Networks for Use-of-Trasnmission-System Charges and Congestion Management",IEEE Power Engineering Society Winter Meeting, January 31- February 4 1999, New York City, New York, USA, pp.799-405.

[41] R.S.Fang, A.K.David, "Optimal Dispatch Under Transmission Contracts", IEEE Transactions on Power Systems, Vol. 14, No. 3, May 1999, pp. 732-737.

[42] A.K.David,"Reconciling Pool and Contract Dospacth in Open Access Transmission Operations", IEE Proceedings on Generation Transmission and Distribution, v145, n 4, July 1998, pp. 468-472.

[43] A.G. Expósito et al., "Fair Allocation of Transmission Power Losses", IEEE Transactions on Power Systems, Vol. 15, No. 1, February 2000, pp. 184-188.

[44] A.J.Wood and B.F.Wollenberg,"Power Generation Operation and Control", New York, NY, WileyInterscience Publication 1996.

[45] R.Reta, A.Vargas, "Electricity Tracing and Loss Allocation Methods Based on Electric Concepts", IEE Proceedings on Generation Transmission and Distribution, v148, n 6, November 2001, pp. 518-522.

[46] Z.Q.Wu, G.Z.Chen,"MVA Power Flow and Loss Analysis for Electricity Market", IEE Proceedings on Generation Transmission and Distribution, v148, n 2, March 2001, pp. 153-158.

[47] A.Fradi, S.Brignone, B.F.Wollenberg, "Calculation of Energy Transaction Allocation Factors", IEEE Transactions on Power Systems, Vol. 16, No. 2, May 2001, pp. 266-271.

[48] A.R.Bhuiya, "Allocation of Transmission Loss in Deregulated Power System Networks", M.Sc. Thesis, Department of Electrical Engineering, University of Saskatchewan, August 1999.

[49] G.Kron, "Tensorial Analysis of Integrated Transmission System -Part II: Off-nominal Turns Ratios", AIEE Transactions, Vol. 71, January 1952, pp. 505-512.

[50] G.Gross, S.Tao, "A Physical-Flow-Based Approach to Allocating Transmission Losses in a Transaction Framework", IEEE Transactions on Power Systems, Vol. 15, No. 2, May 2000, pp. 631-637.

[51] N. Chowdhury, A. Bhuiya, "Determination of Transmission Loss Allocation in a Deregulated Power System Network with Counter-flow", *MEPCON* 2001, Cairo, Egypt, 2002.

[52] A. Bhuiya, N. Chowdhury, "Counter-flow in a Deregulated Power System Network and It's Effect on Transmission Loss Allocation", IEEE Canadian Conference on Electrical and Computer Engineering, Toronto, Ontario, Canada, 2001, pp 431-434.

[53] Wiley Encyclopedia of Electrcial and Electronics Engineering, Vol. 18, John Wiley and Sons, Inc., 1999.

[54] Y.Z.Li, A.K.David, "Wheeling Rates of Reactive Power Flow under marginal Cost Pricing", IEEE Transactions on Power Systems, Vol. 9, No. 3, August 1994, pp. 1263-1269.

[55] J.W.Lamont, J.Fu, "Cost Analysis of Reactive Power Support", IEEE Transactions on Power Systems, Vol. 14, No. 3, August 1999 pp. 890-897.

[56] K.Bhattacharya, J.Zhong, "Reactive Power as an Anicillary Service", IEEE Transactions on Power Systems, Vol. 16, No. 2, May 2001, pp. 294-300.

[57] URL: http://www.caiso.com/docs/2000/04/03/2000040315395119675.pdf

[58] N.A.Chowdhury, “Spinning Reserve Assessment in Interconnected Generation System”, Ph.D. dissertation, Department of Electrical Engineering, University of Saskatchewan, Saskatoon, Canada, 1989.

[59] CL.Tseng et al., “Price-based adaptive spinning reserve requirements in power system scheduling”, International Journal of Electrical Power and Energy Systems, Vol. 21, No. 2, February 1999, pp. 137-145.

[60] Arroyo, J.M., Conejo, A.J., “Optimal Response of a Power Generator to Energy, AGC, and Reserve Pool-based Markets”, ”, IEEE Transactions on Power Systems, Vol. 17, No. 2, May 2002, pp. 404-410.

[61] URL: http://www.esc.ab.ca/pdf/cherylrunge.pdf

[62] URL: http://www.theimo.com/

[63] URL: http://www.iso-ne.com/

[64] URL: http://www.nordpool.no/information/reports/nordic_market_report/chapter_004.html

[65] J. Bialek, D.B.Tam “Tracing the Generators' Output”, IEE International Converence on Opportunites and Advances in Electric Power Generations, Durham, UK, 18-20 March 1996, pp. 133-136.

[66] “IEEE Reliability Test System”, A report prepared by the Reliability Test System Task Force of the Application of Probability Methods Subcommittee, IEEE Transactions on Power Apparatus and Systems, Vol. PAS-98, December 1979, pp 2047-2054.

[67] M.E. El-Hawary, “Electrical Power Systems”, IEEE Press, Power Systems Engineering Series, 1995.

[68] M.E. El-Hawary, G.S. Christensen, “Optimal Operation of Electric Power Systems”, Academic Press Inc., 1979.

[69] E.E.George, "Intrasystem Transmission Losses", AIEE Transactions, Vol. 62, March 1943, pp. 153-158.

[70] A. Bhuiya, N. Chowdhury, "Allocation of Transmission Losses in a Deregulated Power System Network", IEEE Canadian Conference on Electrical and Computer Engineering, Edmonton, Alberta, Canada, pp 1148-1152.

[71] H.E. Brown, "Solution of Large Networks by Matrix Methods", Wiley, Newyork, 1975.

[72] I.J. Nagrath, D.P. Kothari, "Modern Power System Analysis", Tata-McGraw-Hill Publishing Co. Ltd., 1980.

[73] M.E. El-Hawary, "Electrical Energy Systems", CRC Press, 2000.

[74] Shoults R.R., et.al.,"A Practical Approach to Unit Commitment, Economic Dispacth and Savings Allocation for Multiple-Area Pool Operation with Import/Export Constraints", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-99, No.2 March/April 1980, pp 625-633.

[75] Oteng-Adjei J, "Impacts of Energy Limited Units on Generating Capacity Adequacy Assessment", M.Sc. Thesis, Department of Electrical Engineering, University of Saskatchewan, Saskatoon, Canada, 1986.

[76] Oteng-Adjei J, "Impacts of Energy Limited Units on Generating Capacity Adequacy Assessment", Ph.D. dissertation, Department of Electrical Engineering,University of Saskatchewan, Saskatoon, Canada, 1989.

[77] URL: http://www.iso-ne.com/smd/market_rule_1_and_NEPOOL_manuals

[78] R. Billinton, R.N. Allan, "Reliability Evaluation of Power Systems", Plenum Press, New York, 1990.

[79] R. Billinton, W. Li, "Reliability Assessment of Electrical Power Systems Using Monte Carlo Methods", Plenum Press, New York, 1994

APPENDIX - A

The figure and data are obtained from [55, 64, 65, 70, 71].

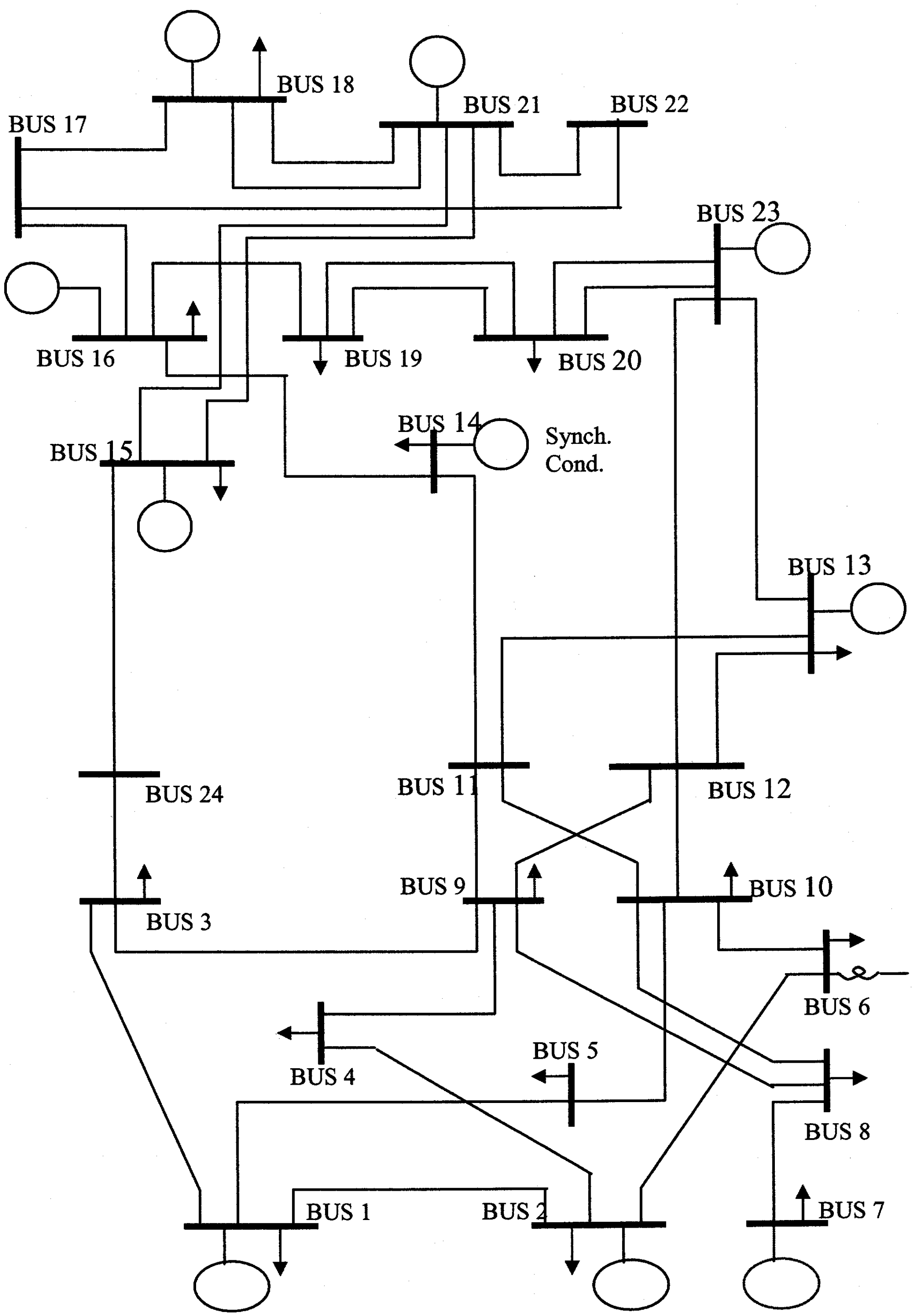

Figure A.1: IEEE 24-Bus Reliability Test System.

Table A1: Line impedance Data.

Line #	From Bus	To Bus	R (p.u.)	X (p.u.)
1	1	2	0.0026	0.0139
2	1	3	0.0546	0.2112
3	1	5	0.0218	0.0845
4	2	4	0.0328	0.1267
5	2	6	0.0497	0.1920
6	3	9	0.0308	0.1190
7	3	24	0.0023	0.0839
8	4	9	0.0268	0.1037
9	5	10	0.0228	0.0883
10	6	10	0.0139	0.0605
11	7	8	0.0159	0.0614
12	8	9	0.0427	0.1651
13	8	10	0.0427	0.1651
14	9	11	0.0023	0.0839
15	9	12	0.0023	0.0839
16	10	11	0.0023	0.0839
17	10	12	0.0023	0.0839
18	11	13	0.0061	0.0476
19	11	14	0.0054	0.0418
20	12	13	0.0061	0.0476
21	12	23	0.0124	0.0966
22	13	23	0.0111	0.0865
23	14	16	0.0050	0.0389
24	15	16	0.0022	0.0173
25	15	21	0.0063	0.0490
26	15	21	0.0063	0.0490
27	15	24	0.0067	0.0519
28	16	17	0.0033	0.0259
29	16	19	0.0030	0.0231

30	17	18	0.0018	0.0144
31	17	22	0.0135	0.1053
32	18	21	0.0033	0.0259
33	18	21	0.0033	0.0259
34	19	20	0.0051	0.0396
35	19	20	0.0051	0.0396
36	20	23	0.0028	0.0216
37	20	23	0.0028	0.0216
38	21	22	0.0087	0.0678

Table A2: Bus data.

Bus #	Bus Type	Bus Voltage	Bus Angle	P_G	Q_G	P_L	Q_L
1	0		0.00			1.08	0.22
2	2		0.00			0.97	0.20
3	1	0.00	0.00			1.80	0.37
4	1	0.00	0.00			0.74	0.15
5	1	0.00	0.00			0.71	0.14
6	1	0.00	0.00			1.36	0.28
7	2		0.00			1.25	0.25
8	1	0.00	0.00			1.71	0.35
9	1	0.00	0.00			1.75	0.36
10	1	0.00	0.00			1.95	0.40
11	1	0.00	0.00			0.00	0.00
12	1	0.00	0.00			0.00	0.00
13	2		0.00			2.65	0.54
14	1	0.00	0.00			1.94	0.39
15	2		0.00			3.17	0.64
16	2		0.00			1.00	0.20
17	1	0.00	0.00			0.00	0.00
18	2		0.00			3.33	0.68

19	1	0.00	0.00			1.81	0.37
20	1	0.00	0.00			1.28	0.26
21	2		0.00			0.00	0.00
22	2		0.00			0.00	0.00
23	2		0.00			0.00	0.00
24	1	0.00	0.00			0.00	0.00

Bus Type 0 = swing bus

Bus Type 1 = load bus

Bus Type 2 = generator bus

Table A3: Generation Data.

Bus	P_{Gmax} (p.u.)	Q_{Gmax} (p.u.)	Q_{Gmin} (p.u.)	V_{max} (p.u.)	V_{min} (p.u.)
1	1.92	1.20	-0.75	1.05	0.95
2	1.92	1.20	-0.75	1.05	0.95
7	3.00	2.70	0.00	1.05	0.95
13	5.91	3.60	0.00	1.05	0.95
15	2.15	1.65	-0.75	1.05	0.95
16	1.55	1.20	-0.75	1.05	0.95
18	4.00	3.00	-0.75	1.05	0.95
21	4.00	3.00	-0.75	1.05	0.95
22	3.00	1.45	-0.90	1.05	0.95
23	6.60	4.50	-1.75	1.05	0.95

Table A4: Generating Unit Reliability Data.

Unit Size (MW)	Number of Units	Forced Outage Rate	MTTF (hrs.)	MTTR (hrs.)	Scheduled Maintenance (wks/year)
12	5	0.02	2940	60	2
20	4	0.10	450	50	2
50	6	0.01	1980	20	2
76	4	0.02	1960	40	3
100	3	0.04	1200	50	3
155	4	0.04	960	40	4
197	3	0.05	950	50	4
350	1	0.08	1150	100	5
400	2	0.12	1100	150	6

MTTF = mean time to failure

MTTR = mean time to repair

$$\text{Forced Outage Rate} = \frac{\text{MTTR}}{\text{MTTR} + \text{MTTF}}$$

Table A5: Generating Unit Locations Data.

Bus	Unit 1 (MW)	Unit 2 (MW)	Unit 3 (MW)	Unit 4 (MW)	Unit 5 (MW)	Unit 6 (MW)
1	20	20	76	76		
2	20	20	76	76		
7	100	100	100			
13	197	197	197			
15	12	12	12	12	12	155
16	155					
18	400					
21	400					
22	50	50	50	50	50	50
23	155	155	350			

Table A6: Generating Unit Cost Data.

Unit Size (MW)	Number of Units	Running Cost Parameters		
		a	b	C
12	5	0.13733	23.27773	30.39611
20	4	0.18256	37.55452	40.0000
50	6	0.00000	0.50000	0.00000
76	4	0.01131	12.14489	100.43962
100	3	0.02203	17.92387	286.24109
155	4	0.00667	9.27063	206.70340
197	3	0.00300	20.02271	301.22318
350	1	0.00392	8.91965	388.25027
400	2	0.00028	5.34515	216.57585

APPENDIX – B

Case 1:

Profit of an IPP is written as

$$\pi = \Phi P + \Phi_1 T + \Psi Q - aP^2 - bP - c - dQ - e \tag{B.1}$$

The Lagrangian can be expressed as:

$$L = \pi - \lambda\left((P+T)^2 + Q^2 + \tau^2 - S^2\right) \tag{B.2}$$

Taking the derivatives of the Lagrangian with respect to real and reactive power, spinning reserve, the slack variable τ and the Lagrangian multiplier λ:

$$\frac{\partial L}{\partial P} = \Phi - 2aP - b - 2\lambda\ (P+T) = 0 \tag{B.3}$$

$$\frac{\partial L}{\partial T} = \Phi_1 - 2\lambda(P+T) = 0 \tag{B.4}$$

$$\frac{\partial L}{\partial Q} = \Psi - 2\lambda Q - d = 0 \tag{B.5}$$

$$\frac{\partial L}{\partial \tau} = -2\lambda\tau = 0 \tag{B.6}$$

$$\frac{\partial L}{\partial \lambda} = (P+T)^2 + Q^2 + \tau^2 - S^2 = 0 \tag{B.7}$$

From Equation (B.6)

$$\lambda\tau = 0$$

If τ is not equal to zero, λ must be equal to zero.

$$\lambda = 0 \tag{B.8}$$

From Equation (B.3), the following can be written

$$P = \frac{\Phi - b}{2a} \tag{B.9}$$

From Equation (B.5) the following can be written:

$$\Psi = d \tag{B.10}$$

Equation (B.10) shows that the price of reactive power is equal to its marginal cost if λ is equal to zero. This situation indicates that the magnitude of reactive power Q has no effect on profit and therefore, the producer may set the production of Q at any level within its limits. The system operator may also ask the producer to provide a certain magnitude of Q in order to maintain a desired voltage profile without having to impose an additional cost burden to the producer.

From Equation (B.4),

$$\Phi_1 = 0 \tag{B.11}$$

This indicates λ will be zero if the price of spinning reserve becomes zero and spinning reserve T has no effect on profit and the producer may set the production level of T at any magnitude within its limits.

Case 2:

In this case Q is considered equal to $Q_{\max}$. Equations (B.3, B.4, B.6, B.7) will be used in this case.

If λ is not equal to zero, τ must be equal to zero.

From Equations (B.3) and (B.4), real power production can be determined as:

$$P = \frac{\Phi - \Phi_1 - b}{2a} \tag{B.12}$$

Spinning reserve can be obtained from Equation (B.7)

$$T = \sqrt{S^2 - Q^2{}_{\max}} - P \tag{B.13}$$

If τ is not equal to zero, λ must be equal to zero.

From Equation (B.3),

$$P = \frac{\Phi - b}{2a} \tag{B.15}$$

From Equation (B.4),

$$\Phi_1 = 0 \tag{B.16}$$

This indicates λ will be zero if the price of spinning reserve becomes zero and spinning reserve T has no effect on profit and therefore, the producer may set the production level of T at any magnitude within its limits.

Case 3:

In this case T is considered equal to $T_{\max}$. Equations (B.3, B.5, B.6, B.7) will be used in this case.

If λ is not equal to zero, τ must be equal to zero.

From Equation (B.5),

$$2\lambda = \frac{\Psi - d}{Q} \tag{B.17}$$

Using Equations (B.3) and (B.17), the following can be written:

$$Q(\Phi - 2aP - b) = (\Psi - d)(P + T_{\max}) \tag{B.18}$$

Equations (B.7) and (B.18) is used to obtain a quartic equation of P as follows:

$$\begin{aligned}&4a^2P^4 + 8a^2(T_{\max} - k)P^3 + \left((\Psi - d)^2 - 4a^2\left(S^2 + 2kT_{\max} - (T_{\max} - k)^2\right)\right)P^2 + \\ &\left(2T_{\max}(\Psi - d) + 8a^2k\left(S^2 - T^2{}_{\max} + kT_{\max}\right)\right)P + \left((\Psi - d)^2 - 4a^2k^2\left(S^2 - T^2{}_{\max}\right)\right) = 0\end{aligned} \tag{B.19}$$

Where,

$$k = \frac{\Phi - b}{2a}$$

P can be obtained from Equation (B.19) and subsequently Q can be obtained from Equation (B.18).

If τ is not equal to zero, λ must be equal to zero.

From Equation (B.3),

$$P = \frac{\Phi - b}{2a} \tag{B.20}$$

From Equation (B.5),

$$\Psi = d \tag{B.21}$$

Equation (B.21) shows that the price of reactive power is equal to its marginal cost if λ is equal to zero. This situation indicates that the magnitude of reactive power Q has no effect on profit and therefore, the producer may set the production level of Q at any magnitude within its limits.

In all other cases optimal magnitudes of P, Q and T can be obtained using equations shown in Chapter 6.

APPENDIX – C

Zero Profit Production Curves for Ten-Minute Spinning Reserve:

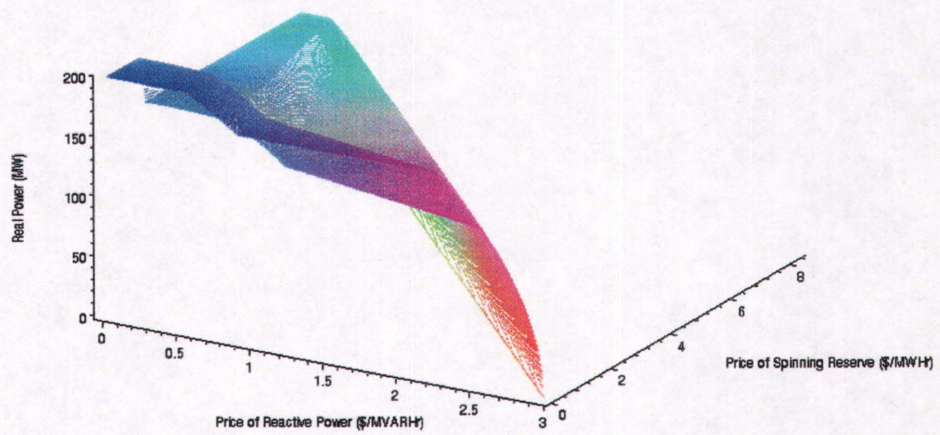

Fig. C.1: Optimum production of real power as a function of reactive power and spinning reserve prices for zero profit condition.

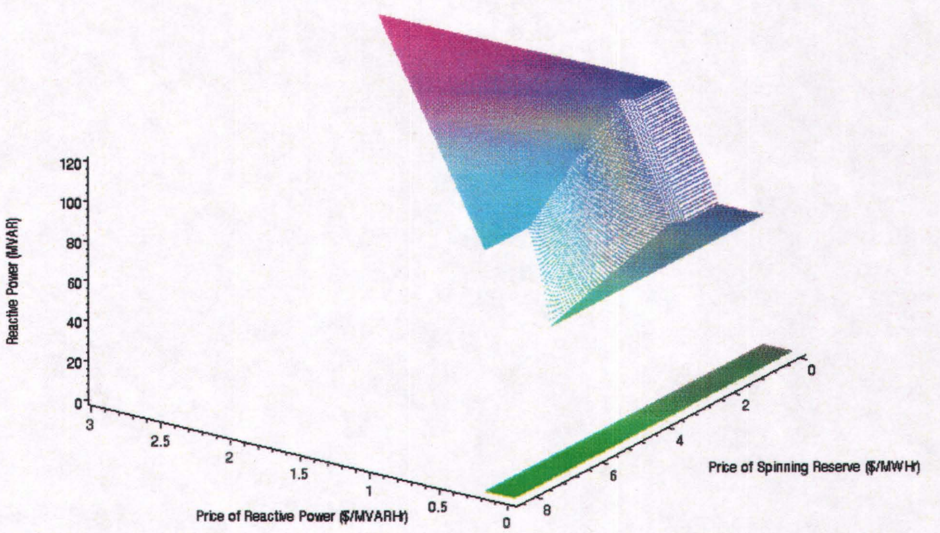

Fig. C.2: Optimum production of reactive power as a function of reactive power and spinning reserve prices for zero profit condition.

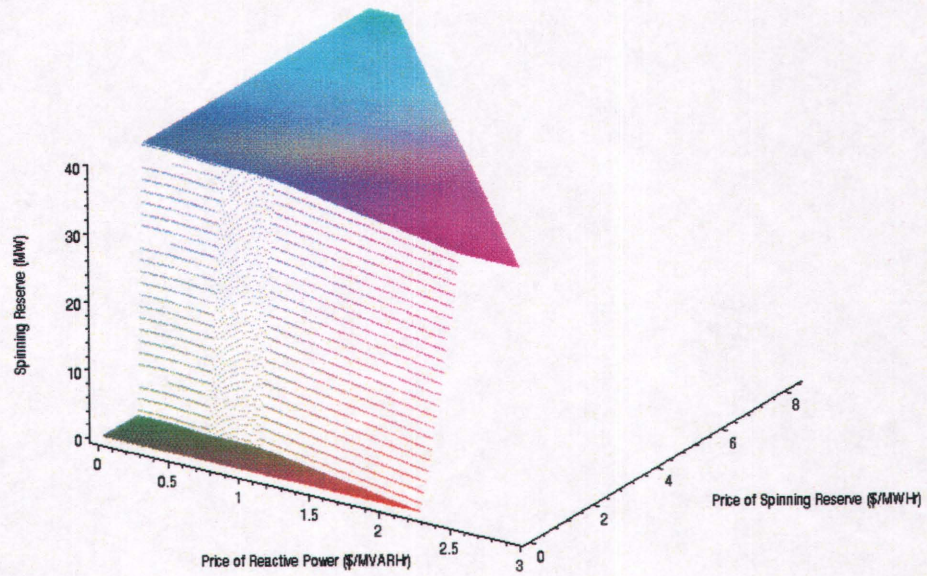

Fig. C.3: Optimum production of spinning reserve as a function of reactive power and spinning reserve prices for zero profit condition.

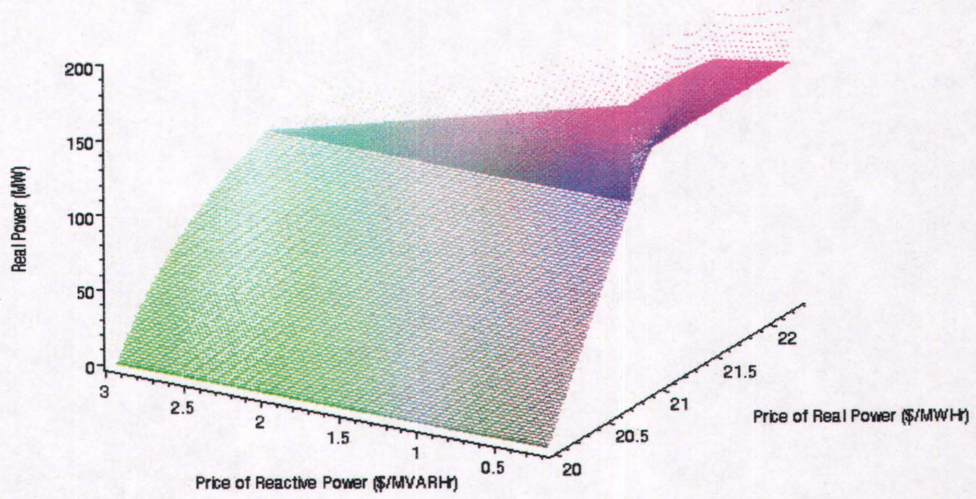

Fig. C.4: Optimum production of real power as a function of reactive power and real power prices for zero profit condition.

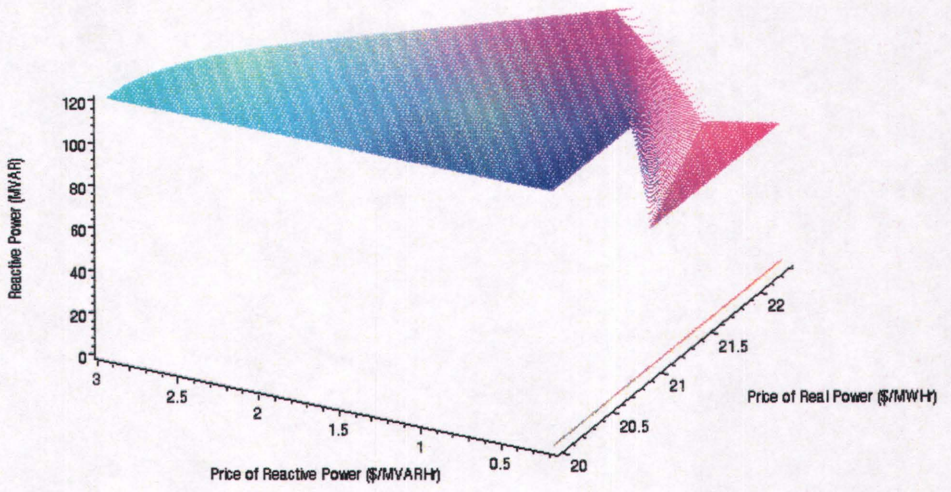

Fig. C.5: Optimum production of reactive power as a function of reactive power and real power prices for zero profit condition.

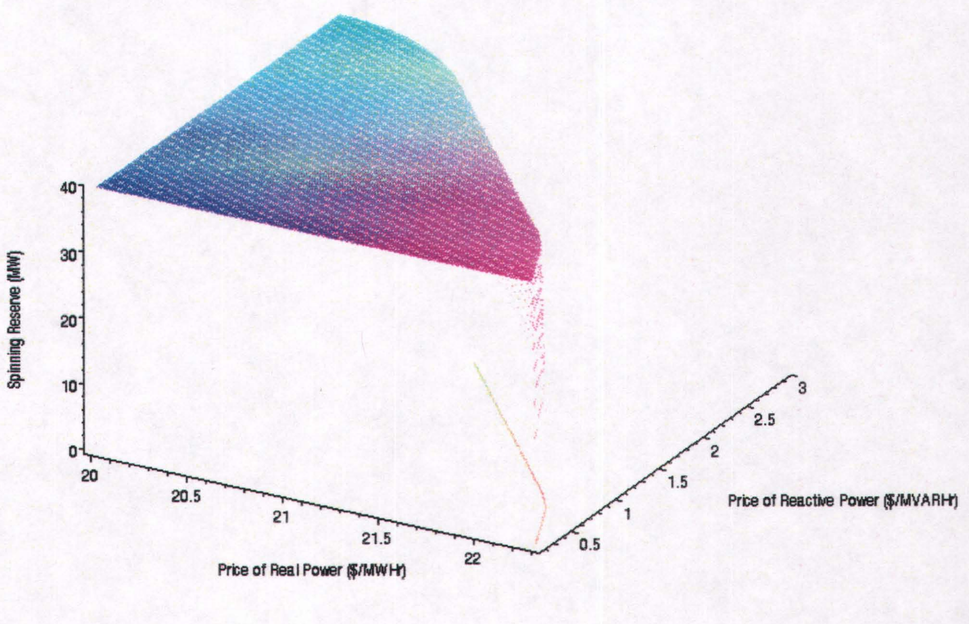

Fig. C.6: Optimum production of spinning reserve as a function of reactive power and real power prices for zero profit condition.

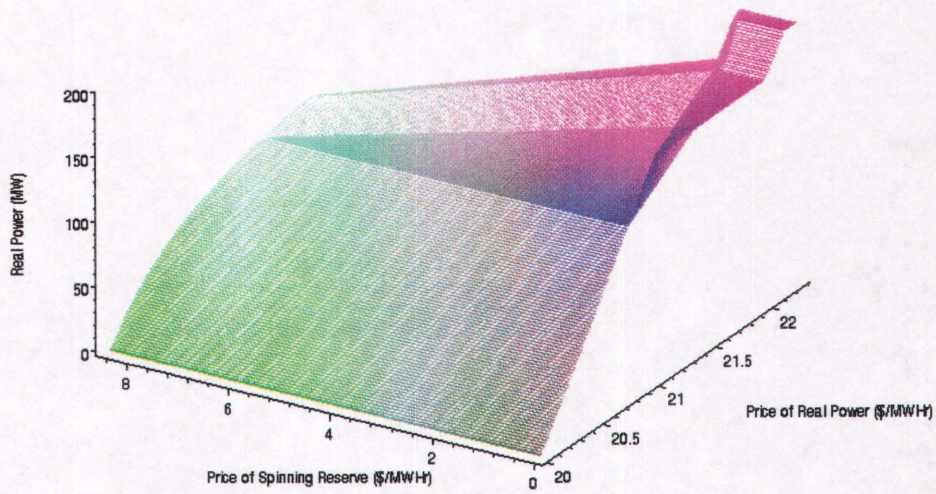

Fig. C.7: Optimum production of real power as a function of spinning reserve and real power prices for zero profit condition.

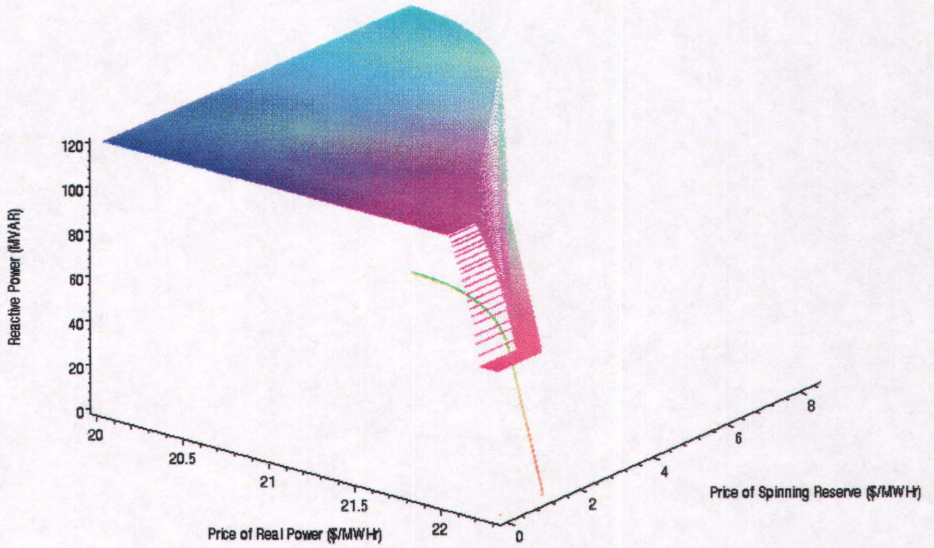

Fig. C.8: Optimum production of reactive power as a function of spinning reserve and real power prices for zero profit condition.

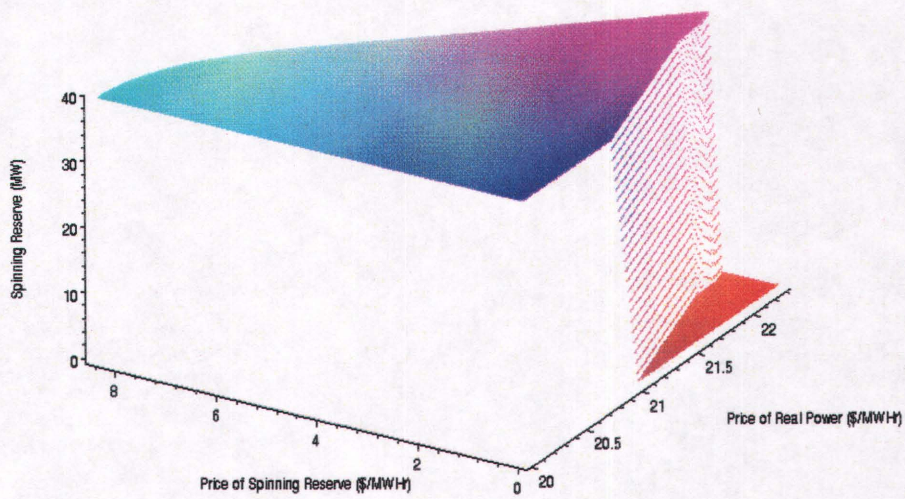

Fig. C.9: Optimum production of spinning reserve power as a function of spinning reserve and real power prices for zero profit condition.

Zero Profit Production Curves for Thirty-Minute Spinning Reserve:

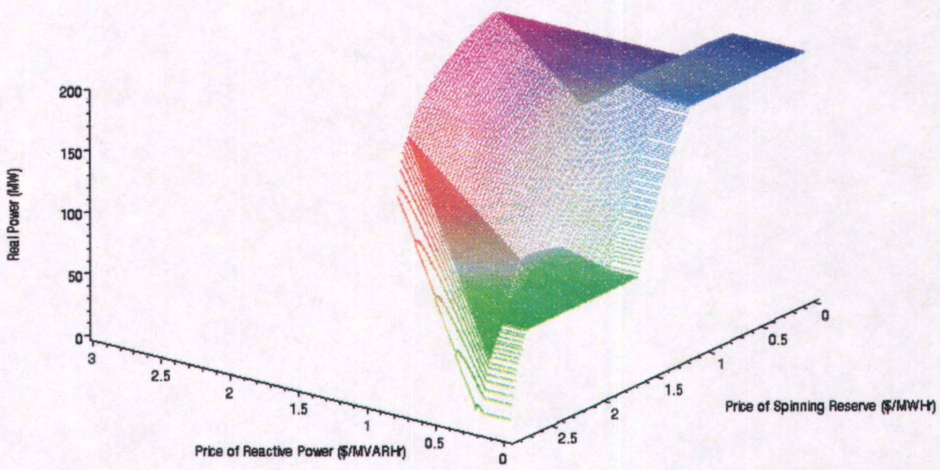

Fig. C.10: Optimum production of real power as a function of reactive power and spinning reserve prices for zero profit condition.

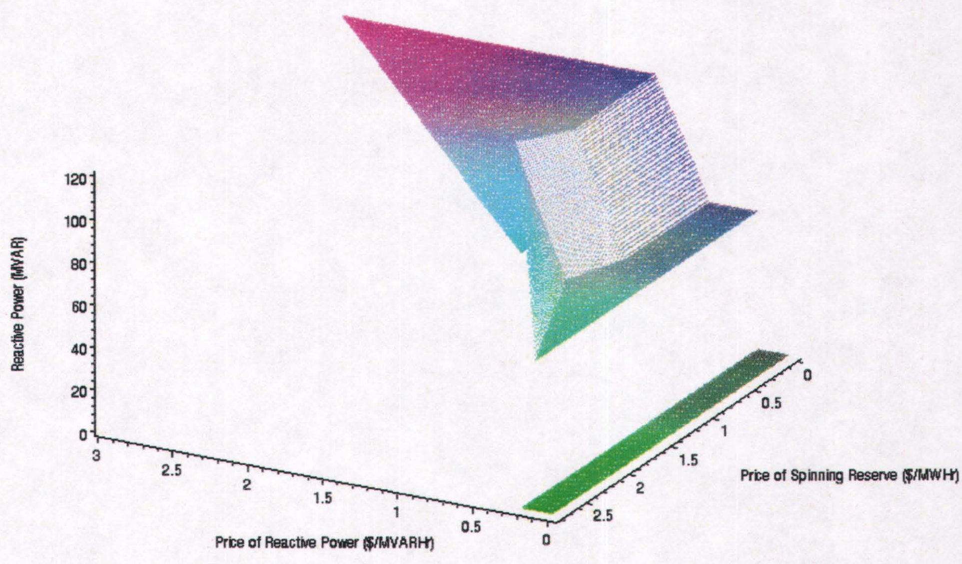

Fig. C.11: Optimum production of reactive power as a function of reactive power and spinning reserve prices for zero profit condition.

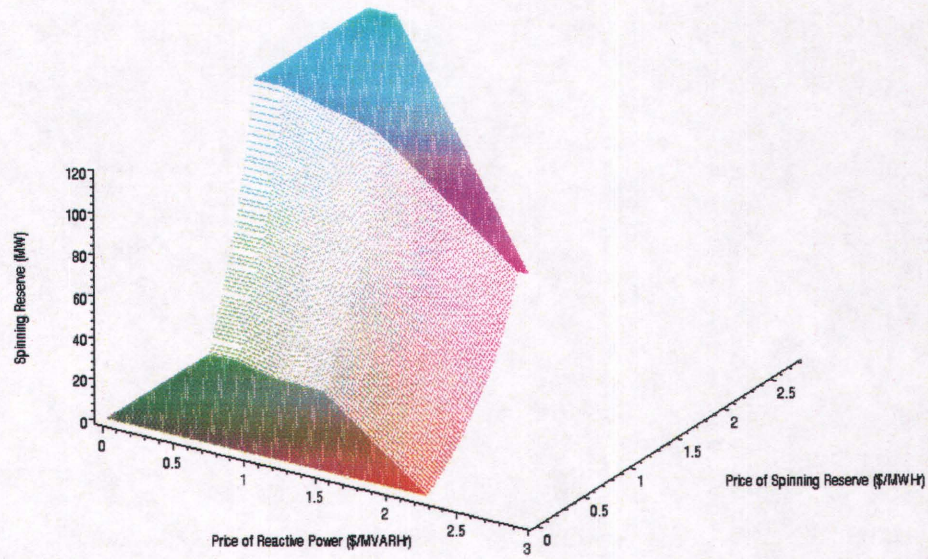

Fig. C.12: Optimum production of spinning reserve as a function of reactive power and spinning reserve prices for zero profit condition.

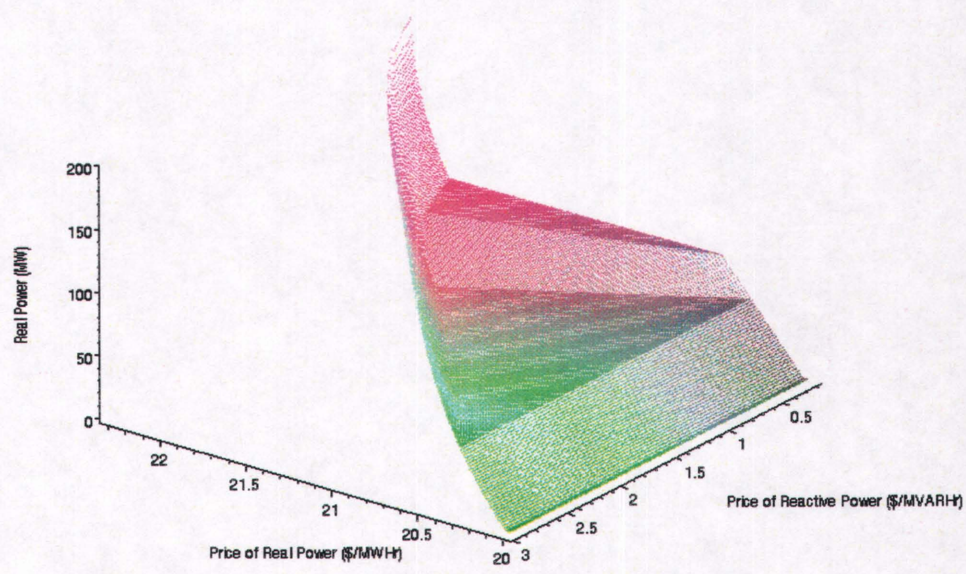

Fig. C.13: Optimum production of real power as a function of reactive power and real power reserve prices for zero profit condition.

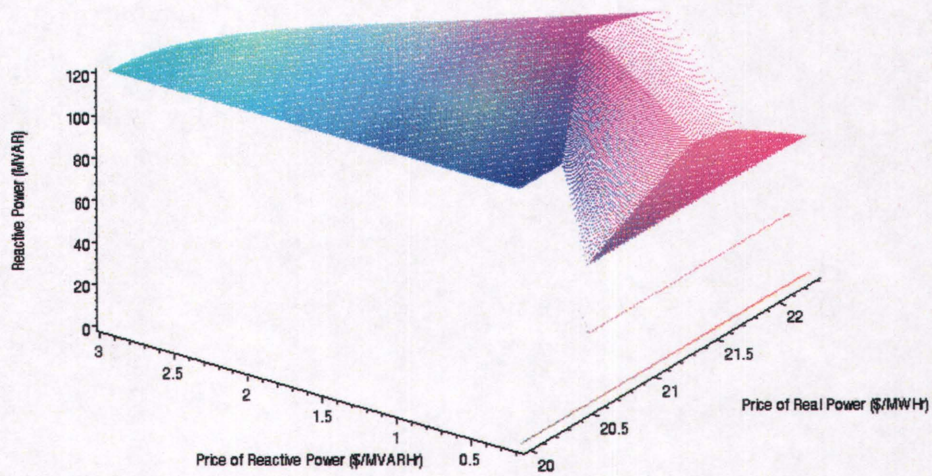

Fig. C.14: Optimum production of reactive power as a function of reactive power and real power prices for zero profit condition.

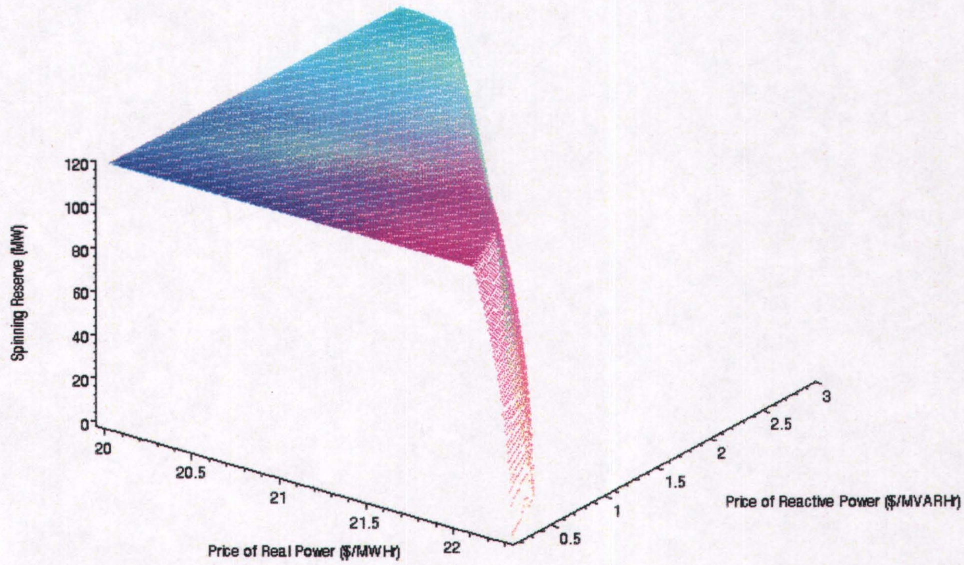

Fig. C.15: Optimum production of spinning reserve power as a function of reactive power and real power prices for zero profit condition.

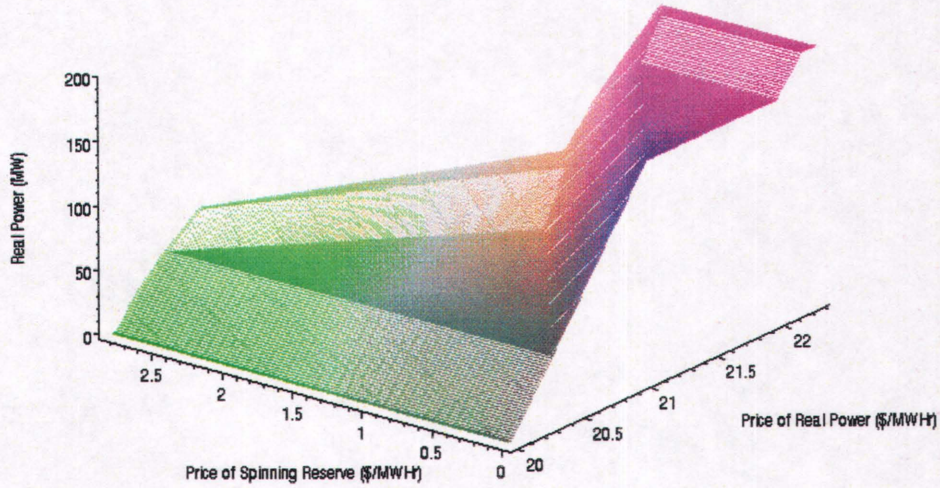

Fig. C.16: Optimum production of real power as a function of real power and spinning reserve prices for zero profit condition.

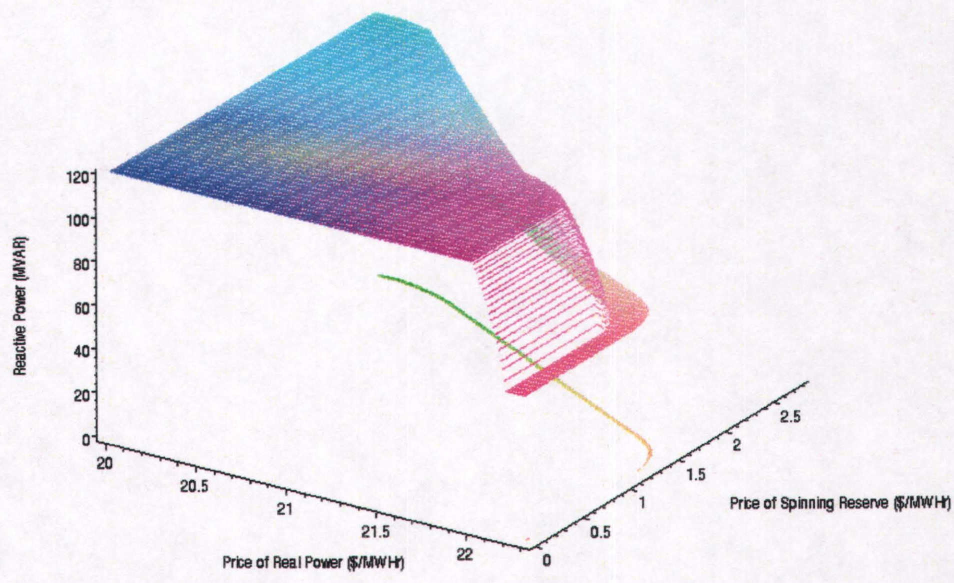

Fig. C.17: Optimum production of reactive power as a function of real power and spinning reserve prices for zero profit condition.

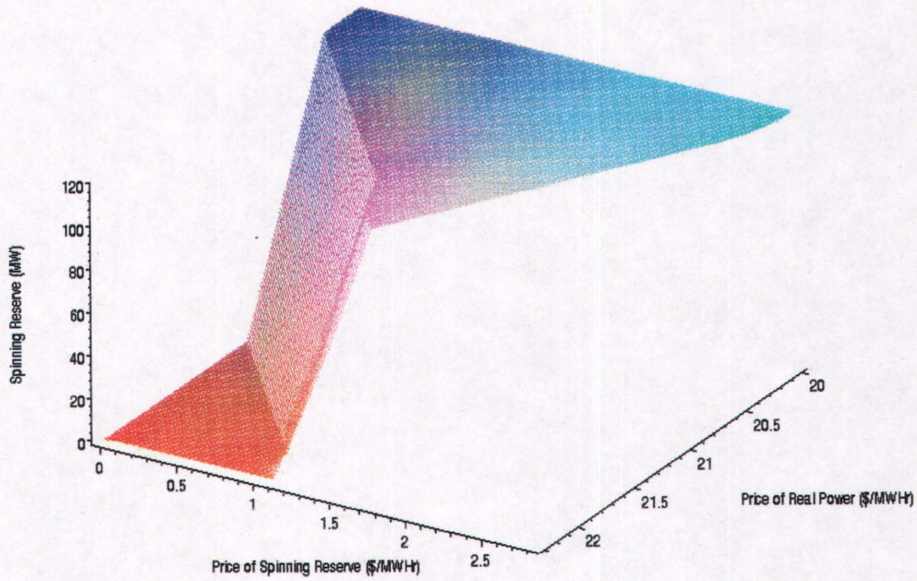

Fig. C.18: Optimum production of spinning reserve as a function of real power and spinning reserve prices for zero profit condition.